

5. [9 points] Jesse has spent the last two years learning Spanish at the Academy of Foreign Languages. He noticed that the number of words in Spanish $S(t)$ he knows is given by the formula

$$S(t) = a \log(80t + 10) - 200$$

where a is a positive constant and t is the number of years after he registered in the Academy.

- a. [3 points] At the end of his first year in the Academy, Jesse knew 300 words in Spanish. What is the **exact** value of a ? Show all your work.

Solution:

$$300 = a \log(80(1) + 10) - 200$$

$$500 = a \log(90)$$

$$a = \frac{500}{\log(90)}.$$

- b. [2 points] Before entering the Academy, Jesse knew some words in Spanish. How many new words has Jesse learned in the Academy during the first two years? Show all your work.

Solution:

$$a = \frac{500}{\log(90)} \approx 255.853.$$

$$S(2) = 255.853 \log(170) - 200 \approx 370.668.$$

$$S(0) = 255.853 - 200 = 55.853.$$

Hence Jesse has learned $S(2) - S(0) \approx 370.668 - 55.853 = 314.815$, 314 new words.

- c. [4 points] Assume Jesse stays in the Academy and he keeps learning new words in Spanish according to the function $S(t)$. How many years does he have to stay in the Academy in order to know 500 words in Spanish? Round your answer to the nearest 0.01 year. Show all your work.

Solution:

$$500 = S(t)$$

$$500 = a \log(80t + 10) - 200$$

$$700 = a \log(80t + 10)$$

$$\frac{700}{a} \approx 2.735 = \log(80t + 10)$$

$$10^{2.735} = 80t + 10$$

$$543.25 = 80t + 10$$

$$t = 6.66 \text{ years.}$$