

7. [13 points] A video is posted on the internet. At 2 pm, the video had 2400 views. By 5 pm, the video had 4000 views. Let $V(t)$ the number of views t hours after noon. Assume that $V(t)$ grows exponentially.
- a. [5 points] Find a formula for $V(t)$. You must find this formula algebraically. All your numbers in your formula should be in **exact** form. Show all your work.

Solution: If $V(t) = ab^t$, then

$$ab^2 = 2400$$

$$ab^5 = 4000$$

Then $b^3 = \frac{5}{3}$. Which yields $b = \left(\frac{5}{3}\right)^{\frac{1}{3}}$. Using

$$ab^2 = 2400$$

$$a \left(\frac{5}{3}\right)^{\frac{2}{3}} = 2400$$

$$a = \frac{2400}{\left(\frac{5}{3}\right)^{\frac{2}{3}}}$$

Hence $V(t) = \frac{2400}{\left(\frac{5}{3}\right)^{\frac{2}{3}}} \left(\frac{5}{3}\right)^{\frac{t}{3}} = 2400 \left(\frac{5}{3}\right)^{\frac{t-2}{3}}$

- b. [2 points] How many views did the video have at noon?

Solution: $V(0) = a = \frac{2400}{\left(\frac{5}{3}\right)^{\frac{2}{3}}} \approx 1707.308$, 1707 views.

- c. [4 points] How long will it take for the video to have 10 thousand views? Round your answer to the nearest 0.01 hour. Show all your work.

Solution: Since $b = \left(\frac{5}{3}\right)^{\frac{1}{3}} \approx 1.185$, then $V(t) = 1707.308(1.185)^t$. In order to be 10 thousand views by time t ,

$$10000 = 1707.308(1.185)^t$$

$$5.857 = (1.185)^t$$

$$\ln(5.857) = t \ln(1.185)$$

$$t = \frac{\ln(5.857)}{\ln(1.185)} \approx 10.413 \text{ hours.}$$

- d. [2 points] What is the continuous growth rate per hour of $V(t)$? Round your answer to the nearest 0.01%.

Solution: $1.185 = e^k$, then $k = \ln(1.185) = .16974$, then the continuous growth rate per hour of $V(t)$ is 16.97% .