

9. [17 points]

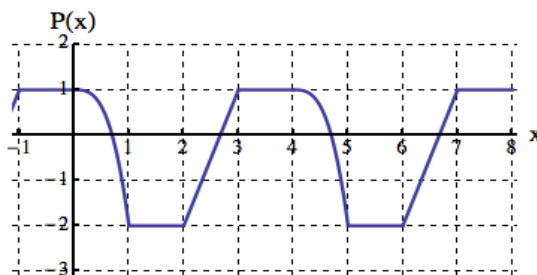
- a. [2 points] Suppose the point $(2, 1)$ is in the graph of $y = V(x)$. What point is in the graph of $H(x) = 9V(x - 5)$?

Solution: Point: $(7, 9)$

- b. [4 points] Suppose that $Q(x)$ has a vertical asymptote at $x = -5$ and a horizontal asymptote at $y = 2$. Find the equation(s) of the vertical and horizontal asymptotes of the function $K(x) = 3 - Q(2x - 1)$.

Solution: Vertical asymptote: $x = -2$ Horizontal asymptote: $y = 1$.

- c. [6 points] The graph of a periodic function $y = P(x)$ is shown below



Consider the periodic function $Q(x) = 4P(\frac{1}{2}x - 1) + 5$. Find the period, the amplitude and the midline of the functions $y = P(x)$ and $y = Q(x)$.

Solution:

- | | | | |
|----------------------------|------------|-----------------------|-----------|
| i) Period of $P(x)$: | 4 | Period of $Q(x)$: | 8. |
| ii) Midline of $P(x)$: | $y = -0.5$ | Midline of $Q(x)$: | $y = 3$. |
| iii) Amplitude of $P(x)$: | 1.5 | Amplitude of $Q(x)$: | 6 |

- d. [5 points] The graph of $y = L(w)$ can be obtained from the graph of $y = e^w$ by doing the following transformations in the given order:

1. Vertical compression by a factor of $\frac{1}{3}$.
2. Horizontal stretch by a factor of 2.
3. Reflection across the y -axis.
4. Horizontal shift to the right by 5.
5. Vertical shift down by 4.

Find a formula for $L(w) =$ _____

Solution: $L(w) = \frac{1}{3}e^{-\frac{1}{2}(w-5)} - 4 = \frac{1}{3}e^{-\frac{1}{2}w + \frac{5}{2}} - 4$