

5. [15 points]

a. [6 points] Consider the function

$$F(x) = \begin{cases} x^2 & \text{for } 0 \leq x \leq 2 \\ 7 + \frac{x}{5} & \text{for } 2 < x \leq 4. \end{cases}$$

Find a piecewise defined formula for the function $G(x) = 3F(x - 1)$.

$$G(x) = \begin{cases} \underline{\hspace{2cm}} & \text{for } \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \text{for } \underline{\hspace{2cm}} \end{cases}$$

b. [5 points] Some of the values of the functions $H(x)$, $K(x)$ and $J(x)$ are shown in the tables below

x	0	1	2	3
$H(x)$	1	-2	3	-4

x	-2	-1	0	1
$J(x)$	2	-1	4	-3

x	0	2	4	6
$K(x)$	-1	2	-3	4

i) The functions $J(x)$ and $K(x)$ are obtained by applying transformations to the function $H(x)$. Find a possible formula for $J(x)$ and $K(x)$. A list of possible answers is shown below. If the answer is not included in the list write your own formula for it in terms of transformations of the function $H(x)$.

$J(x) =$ _____

$K(x) =$ _____

$2H(x - 2)$

$H(x - 2) + 1$

$2H(x + 2)$

$H(x + 2) + 1$

$H(-2x)$

$-H\left(\frac{1}{2}x\right)$

$-H(2x)$

$H\left(-\frac{1}{2}x\right)$

ii) Suppose $H(x)$ is an even function, what is the value of $H(-3)$? _____

c. [4 points] The graph of a function $w = f(z)$ contains the point $(5, -1)$ and has a horizontal asymptote at $w = 2$. Let $g(z) = 1 - f\left(\frac{z}{2} + 3\right)$. Find a point in the graph $g(z)$ and the equation of its horizontal asymptote.

The graph of $g(z)$ contains the point : _____

Equation of the horizontal asymptote of $g(z)$ at _____