2. [11 points] Let $P(t)$ be the average temperature (in °F) in a small moon that rotates around a planet at time $t$ (in hours). Suppose that $P(t)$ is a periodic function with period less than 20 hours. The graph of $y = P(t)$ is shown below.

**Diagram:**

- **a.** [2 points] Find the period of $P(t)$:
  
  $\text{Solution:}$ Period of $P(t) = 16$.

- **b.** [2 points] Find the amplitude of the function $P(t)$:
  
  $\text{Solution:}$ Amplitude $= \frac{50 - (-10)}{2} = 30^\circ F$

- **c.** [2 points] Find the equation of the midline of the function $P(t)$:
  
  $\text{Solution:}$ Midline: $y = \frac{50 + (-10)}{2} = 20$.

- **d.** [3 points] What is the smallest value of $t$ that satisfies $t > 24$ and $P(t) = 30$?
  
  $\text{Solution:}$ The solutions to $P(t) = 30$ for $0 < t < 24$ are (from the graph) $t = 2, 6, 18$ and 22. Hence the next solution is at $t = 18 + 16 = 34$ hours.

- **e.** [2 points] Let $k(t) = 2P(3t)$. What is the period of the function $k(t)$?
  
  $\text{Solution:}$ The graph of $k(t)$ can be obtained from the graph of $P(t)$ by applying a vertical stretch by 2 and a horizontal compression by $\frac{1}{3}$. The only transformation that determines the period of $k(t)$ is the horizontal compression, then the period of $k(t)$ is $\frac{16}{3}$ hours.