5. [15 points]

a. [6 points] Consider the function

$$F(x) = \begin{cases} x^2 & \text{for} \quad 0 \le x \le 2 \\ 7 + \dfrac{x}{5} & \text{for} \quad 2 < x \le 4. \end{cases}$$

Find a piecewise defined formula for the function $G(x) = 3F(x-1)$.

Solution:

$$G(x) = \begin{cases} 3(x-1)^2 & \text{for} \quad 1 \le x \le 3 \\ 3\left(7 + \dfrac{x-1}{5}\right) & \text{for} \quad 3 < x \le 5. \end{cases}$$

b. [5 points] Some of the values of the functions $H(x)$, $K(x)$ and $J(x)$ are shown in the tables below

x	0	1	2	3
$H(x)$	1	-2	3	-4

x	-2	-1	0	1
$J(x)$	2	-1	4	-3

x	0	2	4	6
$K(x)$	-1	2	-3	4

i) The functions $J(x)$ and $K(x)$ are obtained by applying transformations to the function $H(x)$. Find a possible formula for $J(x)$ and $K(x)$. A list of possible answers is shown below. If the answer is not included in the list write your own formula for it in terms of transformations of the function $H(x)$.

Solution: $\mathbf{J(x) = H(x+2) + 1}$ $\qquad$ $\mathbf{K(x) = -H\left(\frac{1}{2}x\right)}$

$2H(x-2)$ $\qquad$ $H(x+2)+1$ $\qquad$ $2H(x+2)$ $\qquad$ $H(x+2)+1$

$H(-2x)$ $\qquad$ $-H\left(\frac{1}{2}x\right)$ $\qquad$ $-H(2x)$ $\qquad$ $H\left(-\frac{1}{2}x\right)$

ii) Suppose $H(x)$ is an even function, what is the value of $H(-3)$?

Solution: $H(-3) = -4$

c. [4 points] The graph of a function $w = f(z)$ contains the point $(5, -1)$ and has a horizontal asymptote at $w = 2$. Let $g(z) = 1 - f\left(\frac{z}{2} + 3\right)$. Find a point in the graph $g(z)$ and the equation of its horizontal asymptote.

Solution: The graph of $g(z)$ contains the point $\mathbf{(4, 2)}$.
Equation of the horizontal asymptote of $g(z)$ at $\mathbf{w = -1}$.