

7. [11 points] Ammonia was spilled into two lakes. Companies A and B are hired to clean the first and second lakes, respectively. Let  $A(t)$  be the amount of ammonia (in gallons) in the first lake  $t$  hours after company A starts cleaning the lake, where

$$A(t) = 2300(0.78)^t.$$

- a. [2 points] How many gallons of ammonia were spilled in the first lake?

*Solution:* Answer: 2300 gallons.

- b. [2 points] What is the continuous rate per hour of the function  $A(t)$ ? Your answer needs to be in exact form.

*Solution:* Using the fact that  $e^k = 0.78$ , we get  $k = \ln(0.78)$ .

- c. [3 points] How long does it take to reduce the amount of ammonia by 20 percent of its initial amount? Show all your work. Your answer needs to be in exact form. Include units.

*Solution:*

$$2300(0.78)^t = 0.8(2300)$$

$$(0.78)^t = 0.8$$

$$\ln((0.78)^t) = \ln(0.8)$$

$$t \ln(0.78) = \ln(0.8) \quad t = \frac{\ln(0.8)}{\ln(0.78)} \text{ hours.}$$

Company B states that their cleaning method is capable of reducing the amount of ammonia in the lake at a continuous rate of 30 percent every hour.

- d. [3 points] Let  $B(t)$  be the amount of ammonia (in gallons) in the second lake  $t$  hours after company B starts cleaning the second lake. Find a formula for  $B(t)$  if the amount of ammonia spilled in the second lake is 3 thousand gallons.

*Solution:*  $B(t) = 3000e^{-0.3t}$

- e. [1 point] Which company reduces the amount of ammonia faster? Circle your answer.

*Solution:* Since the continuous rate per hour of company A,  $k = \ln(0.78) \approx -0.248$  is larger than the continuous rate per hour of company B,  $k = -0.3$ . Then the method from company B reduces the ammonia faster.