8. [13 points] Solve the following equations algebraically. Your solutions should be in **exact form**. Show all your work to receive full credit.

a. [4 points] $\log(2x) - \log(2x+1) = 3$.

Solution:

$$\begin{aligned}
\log(2x) - \log(2x+1) &= 3 \\
\log\left(\frac{2x}{2x+1}\right) &= 3 \\
\frac{2x}{2x+1} &= 10^3 \\
2x &= 1000(2x+1) \\
2x &= 2000x + 1000 \\
-1998x &= 1000 \\
x &= -\frac{1000}{1998}
\end{aligned}$$

No solution.

b. [4 points] $\ln(6e^{2x} - 8) = 2x$

Solution:

$$\begin{aligned}
\ln(6e^{2x} - 8) &= 2x \\
6e^{2x} - 8 &= e^{2x} \\
5e^{2x} &= 8 \\
e^{2x} &= \frac{8}{5} \\
2x &= \ln\left(\frac{8}{5}\right) \qquad x = \frac{1}{2}\ln\left(\frac{8}{5}\right)
\end{aligned}$$

c. [5 points] $1.3(10^x) = (3.4)^{3x}$

Solution:

$$\begin{aligned}
1.3(10^x) &= (3.4)^{3x} \\
\log(1.3(10^x)) &= \log((3.4)^{3x}) \\
\log(1.3) + \log(10^x) &= 3x\log(3.4) \\
\log(1.3) + x &= 3x\log(3.4) \\
\log(1.3) &= 3x\log(3.4) - x \\
\log(1.3) &= x(3\log(3.4) - 1) \\
x &= \frac{\log(1.3)}{3\log(3.4) - 1}.
\end{aligned}$$