

3. [12 points] Let  $C(t)$  and  $A(t)$  be the production (in thousands of pounds) of corn and apples in a farm  $t$  years after 2002, where

$$C(t) = 200e^{-0.4t+1} \quad \text{and} \quad A(t) = 120e^{0.5t}$$

- a. [3 points] What is the annual percent growth rate of the function  $C(t)$ ? Your answer must be exact or accurate up to the first two decimals. Show all your work.

*Solution:* Since  $C(t) = 200e^{-0.4t+1}$ , then  $b = e^{-0.4}$  and  $r = b - 1 = e^{-0.4} - 1 \approx -0.32$ .

- b. [4 points] How long after 2002 will the production of corn be reduced to a third of its size in that year? Your answer must be exact or accurate up to the first two decimals. Show all your work.

*Solution:*

$$200e^{-0.4t+1} = \frac{200}{3}e$$

$$e^{-0.4t} = \frac{1}{3}$$

$$-0.4t = \ln\left(\frac{1}{3}\right) \quad t = -\frac{1}{0.4} \ln\left(\frac{1}{3}\right) \approx 2.74 \text{ years after 2002.}$$

- c. [5 points] According to these functions, when will the production of corn be the same as the production of apples? Your answer must be in **exact form**. Show all your work.

*Solution:*

$$200e^{-0.4t+1} = 120e^{0.5t}$$

$$200e(e^{-0.4t}) = 120e^{0.5t}$$

$$\frac{200e}{120} = \frac{e^{0.5t}}{e^{-0.4t}}$$

$$e^{0.9t} = \frac{5}{3}e$$

$$0.9t = \ln\left(\frac{5}{3}e\right)$$

$$t = \frac{1}{0.9} \ln\left(\frac{5}{3}e\right).$$