

4. [10 points] A new drug, Lexicor, helps reduce the symptoms of the common cold. Doctors recommend to take Lexicor the moment a patient starts showing symptoms of a cold. Let

$$T(x) = 200 - 150 \log(ax + 3)$$

be the length of time (in hours) needed for the drug to eliminate the common cold symptoms after a dose of  $x$  mg. In this problem  $a$  is a nonzero constant.

- a. [2 points] According to the function  $T(x)$ , how long will it take for the symptoms of the common cold to disappear, after a patient starts showing symptoms of a cold, if he does not take Lexicor? *Include units.*

*Solution:*

$$T(0) = 200 - 150 \log(a(0) + 3) = 200 - 150 \log(3) \approx 128.43 \text{ hours.}$$

- b. [4 points] List all the transformations, in order, that you need to apply to the graph of the function  $f(x) = 150 \log(x)$  in order to get the graph of the function  $y = T(x)$ . Assume that  $0 < a < 1$ . Make sure to write each transformation carefully.

*Solution:*

- 1) Horizontal shift to the left by 3.
- 2) Horizontal stretch by  $\frac{1}{a}$ .
- 3) Reflection about the  $x$ -axis.
- 4) Vertical shift up by 200.

- c. [4 points] Find the value of the constant  $a$  if the symptoms of the common cold are eliminated 25 hours after taking a dose of 300 mg of Lexicor. Your answer must be exact or include at least three decimals. Show all your work.

*Solution:*

$$\begin{aligned} 200 - 150 \log(300a + 3) &= 25 \\ -150 \log(300a + 3) &= -175 \\ \log(300a + 3) &= \frac{175}{150} = \frac{7}{6} \\ 300a + 3 &= 10^{\frac{7}{6}} \\ 300a &= 10^{\frac{7}{6}} - 3 \\ a &= \frac{1}{300} \left( 10^{\frac{7}{6}} - 3 \right) \approx 0.0389. \end{aligned}$$