2. [11 points] The number of bees on Percy's uncle's farm has been decreasing over the past five years. The number of bees t years after 2012 on the farm is given by the exponential function

$$B(t) = 7000e^{-0.2t}.$$

a. [3 points] Find the annual decay rate of the bee population in exact form.

The annual decay rate is $e^{-0.2} - 1$.

Solution: If k is the continuous decay rate, and b is the growth factor, the annual decay rate is $r = b - 1 = e^k - 1 = e^{-0.2} - 1$.

b. [4 points] Percy's uncle will need to order more bees when the population of bees falls below 1000. How many years after 2012 will this occur? Give your answer in exact form or accurate to three decimal places.

Percy's uncle will need to order more bees $\dfrac{\ln(1/7)}{-0.2}$ years after 2012.

Solution: Setting $B(t) = 1000$, we get

$$1000 = 7000e^{-0.2t}.$$

If we divide both sides by 7000 and then take ln of both sides, we get

$$\ln(1/7) = -0.2t.$$

So

$$t = \frac{\ln(1/7)}{-0.2}.$$

c. [4 points] The number of mosquitoes on Percy's uncle's farm has been increasing at an annual rate of 9%. Find the doubling time of the mosquito population. Give your answer in exact form or accurate to three decimal places.

The doubling time of the mosquito population is $\dfrac{\ln 2}{\ln 1.09}$ years.

Solution: The mosquito population can be modeled by an exponential function $M(t) = ab^t$ with t in years. If the annual growth rate is 9%, then $b = 1.09$. Using $2a$ for $M(t)$ to find doubling time, we get $2a = a(1.09)^t$. Solving, we get

$$t = \frac{\ln 2}{\ln 1.09}.$$