

4. [11 points] In chemistry, the pH of a substance is a function of the concentration of hydrogen ions per liter of the substance. The pH of a substance with concentration  $C$  hydrogen ions per liter is

$$A(C) = 23.78 - \log(C).$$

- a. [2 points] Lemon juice has a pH of 2.28. What is the concentration of hydrogen ions per liter of lemon juice? Give your answer in exact form.

The concentration of hydrogen ions in lemon juice is  $10^{21.5}$  ions per liter.

*Solution:* Start with  $2.28 = 23.78 - \log(C)$ . Then  $\log(C) = 21.5$ , so  $C = 10^{21.5}$ .

- b. [4 points] If the number of hydrogen ions per liter  $C$  in a substance is doubled, what is the resulting change in pH? Write “increases” or “decreases” in the first blank, and the amount of increase or decrease in the second blank. Give your answer in exact form.

When the concentration of hydrogen ions in a substance doubles, the pH decreases by  $\log(2)$ .

*Solution:*  $A(2C) = 23.78 - \log(2C) = -\log 2 + (23.78 - \log(C)) = -\log 2 + A(C)$ .

- c. [2 points] The owner of the Peter and Sarah’s regular pizza place is looking into canning her pizza sauce to sell in the supermarket. Currently her sauce has  $10^{18}$  hydrogen ions per liter. What is the pH of her sauce? Give your answer in exact form or accurate to three decimal places.

The pH of the sauce is 5.78.

- d. [3 points] The state health department requires that the sauce have a pH lower than 4.7 in order for the sauce to be canned. How many times as many hydrogen ions per liter (compared to the current  $10^{18}$ ) will the sauce need in order for the health department to allow it to be canned? Give a whole number answer that results in a pH above 4 and below 4.7.

The sauce needs 16 times as many ions per liter to be canned.

*Solution:* Each time we double the number of hydrogen ions, the pH of the sauce decreases by  $\log 2 \approx 0.3$  (actually it’s a little more than 0.3) as we saw in part **b**. If we double the number of ions four times for 16 times the original number, the pH will fall by about 1.2 to about 4.58 which is in the range we want.