5. [8 points] Percy is building a zipline from the roof of his uncle’s barn to the base of the farmhouse. The roof of the barn is 20 feet off of the ground. Looking at a 32 degree angle above the ground, he can see the roof of the farmhouse from the ground at the base of the barn. The line from the roof of the barn to the base of the farmhouse makes a 53 degree angle with the side of the barn. The situation is pictured below.

![Diagram of zipline setup](image)

a. [3 points] Find \( R \), the distance from the roof of the barn to the base of the farmhouse. Express your answer in exact form.

\[
R = \frac{20}{\cos 53}.
\]

**Solution:** \( \cos 53 = \frac{20}{R} \), so \( R = \frac{20}{\cos 53} \).

b. [5 points] Find \( H \), the height of the farmhouse. Express your answer in exact form.

*(Hint: You may want to find the distance between the bases of the buildings first)*

\[
H = \frac{20 \tan 53}{\tan 58}.
\]

**Solution:** \( D \), the distance between the buildings satisfies both

\[
\frac{D}{20} = \tan 53
\]

and

\[
\frac{D}{H} = \tan 58.
\]

Solving for \( D \) in both and setting them equal gives us \( 20 \tan 53 = H \tan 58 \). So

\[
H = \frac{20 \tan 53}{\tan 58}.
\]