7. [8 points] At Peter and Sarah's regular pizza place, the pizza is 50 degrees Fahrenheit when it goes into the oven. The oven is 800 degrees Fahrenheit, so a pizza left in the oven will reach 800 degrees after a long time. After 6 minutes in the oven, the pizza is 200 degrees. The temperature of the pizza in degrees Fahrenheit after t minutes in the oven is a function of the form $P(t) = A + Be^{kt}$ with $k < 0$. Find the values of A, B and k in exact form. Show all of your work.

$A =$ 800

$B =$ -750

$k =$ $(\ln 4/5)/6$

Solution: When t is very large, Be^{kt} is very small, and $P(t)$ approaches 800, so $A = 800$.

$$P(0) = 50 = A + B,$$

so $B = -750$. Finally, we use the point $(6, 200)$ to get

$$200 = 800 - 750e^{6k}.$$

This means $4/5 = e^{6k}$, so $k = (\ln 4/5)/6$.

8. [7 points] The temperature in degrees Fahrenheit of the lasagna at the pizza place t minutes after it comes out of the oven is $L(t) = 75 + 225(0.9)^t$.

a. [2 points] What is the air temperature in the pizza place?

The air temperature in the pizza place is 75 degrees F .

b. [2 points] What is the temperature of the lasagna immediately after it comes out of the oven?

The temperature of the lasagna immediately after it comes out of the oven is 300 degrees F.

c. [3 points] How long after the lasagna comes out of the oven does it reach perfect eating temperature of 150 degrees Fahrenheit? Give your answer in exact form or accurate to three decimal places.

The lasagna reaches 150 degrees $\dfrac{\ln(1/3)}{\ln(0.9)}$ minutes after it comes out of the oven.

Solution: We start by setting

$$150 = 75 + 225(0.9)^t.$$

This means $(0.9)^t = 1/3$, so $t = \dfrac{\ln(1/3)}{\ln(0.9)}$.