1. [12 points] In each of the following equations, solve for all possible values of $x$. Be sure to show your work and write your final answer in the blank in exact form. If there are no solutions, write "no solutions" in the blank.
a. [4 points] $\ln \left(2 e^{x}-5\right)=x$.

$$
x=\quad \ln 5
$$

Solution: Taking $e$ to both sides, we get $2 e^{x}-5=e^{x}$. After combining terms we have $e^{x}=5$, so $x=\ln 5$.
b. [4 points] $e^{x+8}=2^{7 x-6}$.

$$
x=\frac{8+6 \ln 2}{7 \ln 2-1}
$$

Solution: Applying ln to both sides and using properties of logs, we have

$$
x+8=(7 x-6) \ln 2 .
$$

If we combine like terms, we get

$$
(7 \ln 2) x-x=8+6 \ln 2 .
$$

Factoring $x$ out of the left hand side of the equation and dividing by what remains, we have that

$$
x=\frac{8+6 \ln 2}{7 \ln 2-1} .
$$

c. [4 points] $\log \left(2 x^{2}-1\right)=0$.

$$
x=\quad \pm 1
$$

Solution: Taking 10 to both sides, we get $2 x^{2}-1=1$. So $2 x^{2}=2$, and $x= \pm 1$.

