

6. [7 points] Kiki is experimenting with a highly radioactive substance, Isotope-Z. Isotope-Z decays at a (non-continuous) rate of 20% per day.
- a. [3 points] Find the continuous decay rate of Isotope-Z. Give your answer as a **percentage** estimated to three decimal places or in exact form.

The continuous decay rate of Isotope-Z is  $\underline{-100 \ln(0.8)}$ .

*Solution:* The daily growth factor for this substance is 0.8. The continuous decay rate is the natural log of the growth factor, so the continuous decay rate is  $\ln(0.8)$ . We multiply by 100 to make it a percentage, and we can leave it negative ( $\ln(0.8) < 0$ ) or make it positive by adding a minus sign.

- b. [4 points] Find the amount of time it takes for Isotope-Z to decrease to 60% of the initial amount present. Do not assume the initial amount is any specific number. Be sure to show all your work and leave your answer in **exact** form with units.

The amount of time is  $\underline{\frac{\ln(0.6)}{\ln(0.8)}}$ .

*Solution:* If we call the initial amount  $A$ , we can set  $0.6A = A(0.8)^t$ . Then  $0.6 = (0.8)^t$ . Taking  $\ln$  on both sides gives us  $\ln(0.6) = t \ln(0.8)$  or  $t = \frac{\ln(0.6)}{\ln(0.8)}$ .