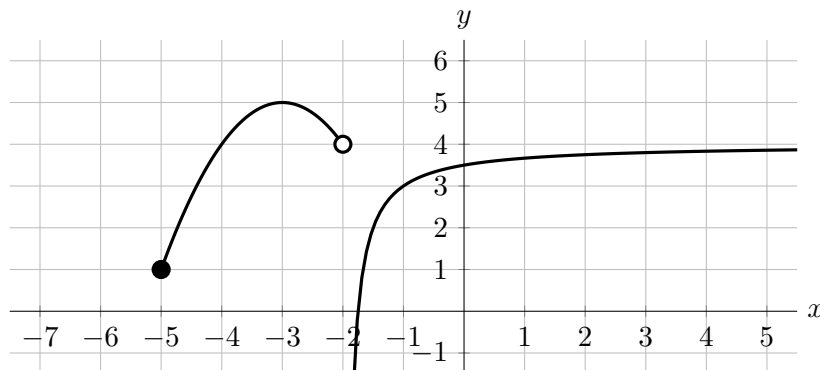


3. [14 points] Consider the function  $f(x)$ , graphed below. Note that  $f(x)$  has one vertical asymptote and one horizontal asymptote, and  $f(x)$  is not defined for  $x$  values to the left of those shown in the graph.



- a. [7 points] Find:
- the domain of  $f(x)$
  - the range of  $f(x)$
  - an equation for the horizontal asymptote of  $f(x)$
  - an equation for the vertical asymptote of  $f(x)$

*Solution:*

- $[-5, -2) \cup (-2, \infty)$
- $(-\infty, 5]$
- $y = 4$
- $x = -2$

- b. [7 points] Let  $g(x) = 3f(-4(x - 2)) + 1$ . Find the following. Show how you obtained your answers, either by showing work, drawing diagrams, or explaining your reasoning.
- the domain of  $g(x)$
  - an equation for the horizontal asymptote of  $g(x)$
  - an equation for the vertical asymptote of  $g(x)$

*Solution:* The graph of  $g(x)$  can be found from the graph of  $f(x)$  using the following transformations:

- a horizontal contraction by a factor of  $1/4$
- a reflection across the  $y$ -axis
- a horizontal shift 2 to the right
- a vertical stretch by a factor of 3
- a vertical shift up by 1.

There are several different orders in which these transformations can be applied, but the stretch/compress and reflection in each direction must be applied before the corresponding shift.

- (i) We can look at how the transformation affects each of endpoints in the domain from part **a**. The domain is only affected by the first three transformations.
- First, compress  $[-5, -2) \cup (-2, \infty)$  by a factor of  $1/4$ : this gives us  $[-5/4, -1/2) \cup (-1/2, \infty)$
  - Then reflect across the  $y$ -axis. This sends each point  $(x, y)$  to the point  $(-x, y)$ . Note that this means we must reverse the order in which the endpoints show up in the interval:  $(-\infty, 1/2) \cup (1/2, 5/4]$ .
  - Finally, shift 2 to the left:  $(-\infty, 5/2) \cup (5/2, 13/4]$ .

This gives a final answer of  $(-\infty, 5/2) \cup (5/2, 13/4]$ .

Another way to find these solutions would be to set  $-5 \leq -4(x - 2) < -2$  and solve for  $x$ . Again, it is important to remember that multiplying by  $-1$  will reverse the order of the inequalities, and that  $+\infty$  will be transformed to  $-\infty$ .

- (ii) The horizontal asymptote to  $f(x)$  is the line  $y = 4$ . Since this line corresponds to the variable on the vertical axis, it is affected by the vertical transformations. We first multiply by 3 and then add 1, giving  $y = 13$ .
- (iii) The vertical asymptote  $x = -2$  for  $f(x)$  will be transformed by the horizontal transformations. Multiplying by  $-1/4$  and then adding 2 gives  $x = 5/2$ . Note that we can also see this in our answer for the domain of  $g(x)$ , where  $5/2$  was not included in the domain.

4. [11 points] Mia and Jonathan sell vegetables at the farmer's market at different booths. Their revenues, in **hundreds** of dollars,  $h$  hours after 9 am on a particular day are  $M(h)$  (for Mia's revenue) and  $J(h)$  (for Jonathan's revenue). Assume that the two functions are invertible.
- a. [2 points] Give a practical interpretation of the equation  $J(2) = 3$ .

*Solution:* This means that Jonathan's revenue at 11 am is equal to \$300.

- b. [3 points] Give a practical interpretation of the expression  $J(M^{-1}(4))$ , or explain why the expression does not make sense in the context of the problem.

*Solution:*  $J(M^{-1}(4))$  is Jonathan's revenue, in hundreds of dollars, at the time when Mia's revenue is \$400.

- c. [3 points] Write an equation corresponding to the following statement: Mia's revenue at 12pm is \$100 less than twice Jonathan's revenue at 11 am.

*Solution:*  $M(3) = 2J(2) - 1$ .