Solution: The graph of $g(x)$ can be found from the graph of $f(x)$ using the following transformations:

- a horizontal contraction by a factor of $1/4$
- a reflection across the y-axis
- a horizontal shift 2 to the right
- a vertical stretch by a factor of 3
- a vertical shift up by 1.

There are several different orders in which these transformations can be applied, but the stretch/compress and reflection in each direction must be applied before the corresponding shift.

(i) We can look at how the transformation affects each of endpoints in the domain from part **a.** The domain is only affected by the first three transformations.
   - First, compress $[-5,-2)\cup(-2,\infty)$ by a factor of $1/4$: this gives us $[-5/4,-1/2)\cup(-1/2,\infty)$
   - Then reflect across the y-axis. This sends each point (x, y) to the point $(-x, y)$. Note that this means we must reverse the order in which the endpoints show up in the interval: $(-\infty, 1/2)\cup(1/2, 5/4]$.
   - Finally, shift 2 to the left: $(-\infty, 5/2)\cup(5/2, 13/4]$.

   This gives a final answer of $(-\infty, 5/2)\cup(5/2, 13/4]$.
   Another way to find these solutions would be to set $-5 \leq -4(x-2) < -2$ and solve for x. Again, it is important to remember that multiplying by -1 will reverse the order of the inequalities, and that $+\infty$ will be transformed to $-\infty$.

(ii) The horizontal asymptote to $f(x)$ is the line $y = 4$. Since this line corresponds to the variable on the vertical axis, it is affected by the vertical transformations. We first multiply by 3 and then add 1, giving $y = 13$.

(iii) The vertical asymptote $x = -2$ for $f(x)$ will be transformed by the horizontal transformations. Multiplying by $-1/4$ and then adding 2 gives $x = 5/2$. Note that we can also see this in our answer for the domain of $g(x)$, where $5/2$ was not included in the domain.

4. [11 points] Mia and Jonathan sell vegetables at the farmer's market at different booths. Their revenues, in **hundreds** of dollars, h hours after 9 am on a particular day are $M(h)$ (for Mia's revenue) and $J(h)$ (for Jonathan's revenue). Assume that the two functions are invertible.

a. [2 points] Give a practical interpretation of the equation $J(2) = 3$.

Solution: This means that Jonathan's revenue at 11 am is equal to \$300.

b. [3 points] Give a practical interpretation of the expression $J(M^{-1}(4))$, or explain why the expression does not make sense in the context of the problem.

Solution: $J(M^{-1}(4))$ is Jonathan's revenue, in hundreds of dollars, at the time when Mia's revenue is \$400.

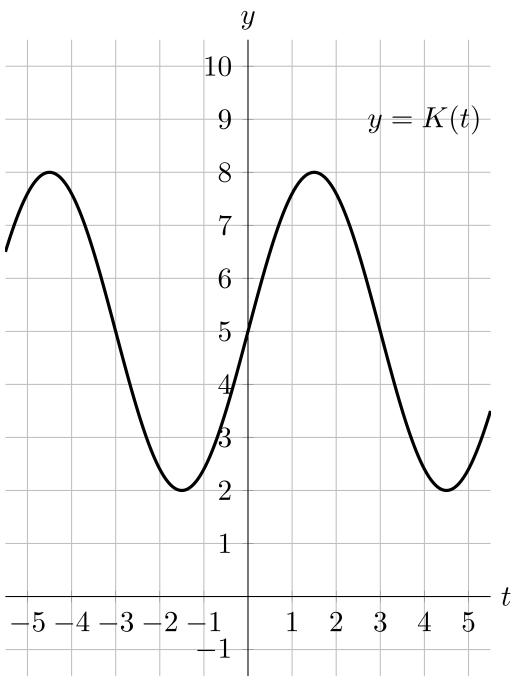
c. [3 points] Write an equation corresponding to the following statement: Mia's revenue at 12pm is \$100 less than twice Jonathan's revenue at 11 am.

Solution: $M(3) = 2J(2) - 1$.

d. [3 points] Let $T(k)$ be the total revenue, in **dollars** of both Mia and Jonathan k **minutes** after 9 am. Find a formula for $T(k)$ in terms of M and/or J.

Solution: $T(k) = 100(M(k/60) + J(k/60))$.

5. [12 points] The graph of a sinusoidal function $y = K(t)$ is given below.



a. [7 points] Find the following.

(i) The amplitude of $K(t)$.
(ii) The midline of $K(t)$.
(iii) The period of $K(t)$.
(iv) A formula for $K(t)$.

Solution:

(i) $(8 - 2)/2 = 3$
(ii) $y = 5$
(iii) 6
(iv) Note that the function starts at its midline at $t = 0$ and increases to the right, so we can use $\sin(t)$ without a horizontal shift or a reflections. Using the values we've found above, we get $3\sin((2\pi/6)t) + 5$.

b. [5 points] Find the first **three** positive values of t for which $K(t) = 7$. Give your answer in exact form.

Solution: We want to know when

$$3\sin((2\pi/6)t) + 5 = 7.$$

Subtracting 5 from both sides, dividing by 3, and taking arcsin, we find

$$(2\pi/6)t = \arcsin(2/3)$$

which gives us

$$t = \frac{3}{\pi}\arcsin(2/3)$$

This is the first solution. We can find the second by using symmetries:

$$t = 3 - \frac{3}{\pi}\arcsin(2/3).$$

The third can be found by adding one period to the first solution:

$$t = 6 + \frac{3}{\pi}\arcsin(2/3).$$