Solution: The graph of \( g(x) \) can be found from the graph of \( f(x) \) using the following transformations:

- a horizontal contraction by a factor of \( 1/4 \)
- a reflection across the \( y \)-axis
- a horizontal shift 2 to the right
- a vertical stretch by a factor of 3
- a vertical shift up by 1.

There are several different orders in which these transformations can be applied, but the stretch/compress and reflection in each direction must be applied before the corresponding shift.

(i) We can look at how the transformation affects each of endpoints in the domain from part \( a \). The domain is only affected by the first three transformations.

- First, compress \([-5, -2) \cup (-\infty, \infty)\] by a factor of \( 1/4 \): this gives us \([-5/4, -1/2) \cup (-1/2, \infty)\).
- Then reflect across the \( y \)-axis. This sends each point \((x, y)\) to the point \((-x, y)\).
  Note that this means we must reverse the order in which the endpoints show up in the interval: \((-\infty, 1/2) \cup (1/2, 5/4)\).
- Finally, shift 2 to the left: \((-\infty, 5/2) \cup (5/2, 13/4)\).

This gives a final answer of \((-\infty, 5/2) \cup (5/2, 13/4)\).

Another way to find these solutions would be to set \(-5 \leq -4(x - 2) < -2\) and solve for \( x \). Again, it is important to remember that multiplying by \(-1\) will reverse the order of the inequalities, and that \(+\infty\) will be transformed to \(-\infty\).

(ii) The horizontal asymptote to \( f(x) \) is the line \( y = 4 \). Since this line corresponds to the variable on the vertical axis, it is affected by the vertical transformations. We first multiply by 3 and then add 1, giving \( y = 13 \).

(iii) The vertical asymptote \( x = -2 \) for \( f(x) \) will be transformed by the horizontal transformations. Multiplying by \(-1/4\) and then adding 2 gives \( x = 5/2 \). Note that we can also see this in our answer for the domain of \( g(x) \), where \( 5/2 \) was not included in the domain.

4. [11 points] Mia and Jonathan sell vegetables at the farmer’s market at different booths. Their revenues, in hundreds of dollars, \( h \) hours after 9 am on a particular day are \( M(h) \) (for Mia’s revenue) and \( J(h) \) (for Jonathan’s revenue). Assume that the two functions are invertible.

a. [2 points] Give a practical interpretation of the equation \( J(2) = 3 \).

Solution: This means that Jonathan’s revenue at 11 am is equal to $300.

b. [3 points] Give a practical interpretation of the expression \( J(M^{-1}(4)) \), or explain why the expression does not make sense in the context of the problem.

Solution: \( J(M^{-1}(4)) \) is Jonathan’s revenue, in hundreds of dollars, at the time when Mia’s revenue is $400.

c. [3 points] Write an equation corresponding to the following statement: Mia’s revenue at 12pm is $100 less than twice Jonathan’s revenue at 11 am.

Solution: \( M(3) = 2J(2) - 1 \).
d. [3 points] Let $T(k)$ be the total revenue, in dollars of both Mia and Jonathan $k$ minutes after 9 am. Find a formula for $T(k)$ in terms of $M$ and/or $J$.

**Solution:** $T(k) = 100(M(k/60) + J(k/60))$.

5. [12 points] The graph of a sinusoidal function $y = K(t)$ is given below.

![Graph of $y = K(t)$](image)

**a.** [7 points] Find the following.
(i) The amplitude of $K(t)$.
(ii) The midline of $K(t)$.
(iii) The period of $K(t)$.
(iv) A formula for $K(t)$.

**Solution:**
(i) $(8 - 2)/2 = 3$
(ii) $y = 5$
(iii) 6
(iv) Note that the function starts at its midline at $t = 0$ and increases to the right, so we can use $\sin(t)$ without a horizontal shift or a reflections. Using the values we’ve found above, we get $3\sin((2\pi/6)t) + 5$.

**b.** [5 points] Find the first three positive values of $t$ for which $K(t) = 7$. Give your answer in exact form.

**Solution:** We want to know when

$$3\sin((2\pi/6)t) + 5 = 7.$$

Subtracting 5 from both sides, dividing by 3, and taking arcsin, we find

$$(2\pi/6)t = \arcsin(2/3)$$

which gives us

$$t = \frac{3}{\pi} \arcsin(2/3).$$

This is the first solution. We can find the second by using symmetries:

$$t = 3 - \frac{3}{\pi} \arcsin(2/3).$$

The third can be found by adding one period to the first solution:

$$t = 6 + \frac{3}{\pi} \arcsin(2/3).$$