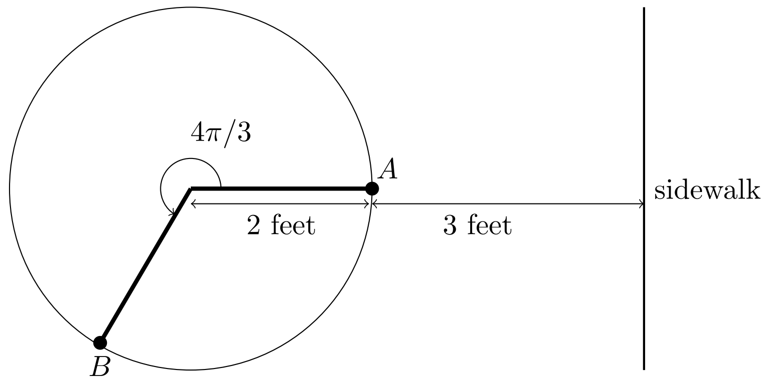
6. [11 points] A duck in swimming in circles along the outer edge of a circular fountain in a park. The duck is 2 feet from the center of the fountain and swimming at a constant speed in a counter-clockwise direction. There is a sidewalk running north-south that passes 3 feet away from the fountain, as shown in the diagram below (which may not be drawn to scale). The duck starts at point A that is closest to the sidewalk. After 4 seconds, the duck is at the point B.



a. [2 points] How long does it take for the duck to make one full lap around the fountain? Include units.

Solution: $4\pi/3$ is $2/3$ of one full rotation, so a lap is $4/(2/3) = 4 \cdot 3/2 = 6$ seconds.

b. [3 points] How far did the duck travel along the circumference of the fountain between point A and point B? Give your answer in exact form and include units.

Solution: We know this is an arc with angle $\theta = 4\pi/3$ and radius is 2 feet, so the corresponding arc length is $r\theta = 2 \cdot 4\pi/3 = 8\pi/3$.

c. [6 points] Find a function $D(t)$ that gives the (horizontal) distance in feet between the duck and the sidewalk t seconds after the duck starts swimming.

Solution: From part (a), we know that this function will be periodic with period 6.
If we imagine a coordinate system with the origin at the center of the circle, then the x-coordinate of the duck's location is given by $2\cos(2\pi/6t)$, and so the distance from the sidewalk, at $x = 5$, to the duck, is found by taking the difference: $5 - 2\cos(2\pi/6t)$.
Another approach would be to note that since the duck is moving in a circle, this distance is a sinusoidal function moving between a minimum of 3 when $t = 0$ and a maximum of $3 + 2 + 2 = 7$. This means that the amplitude is 2 and the midline value is 5, and since the function starts at a minimum, we can use $-\cos(t)$ without a horizontal shift. This gives us $-2\cos(2\pi/6t) + 5$.