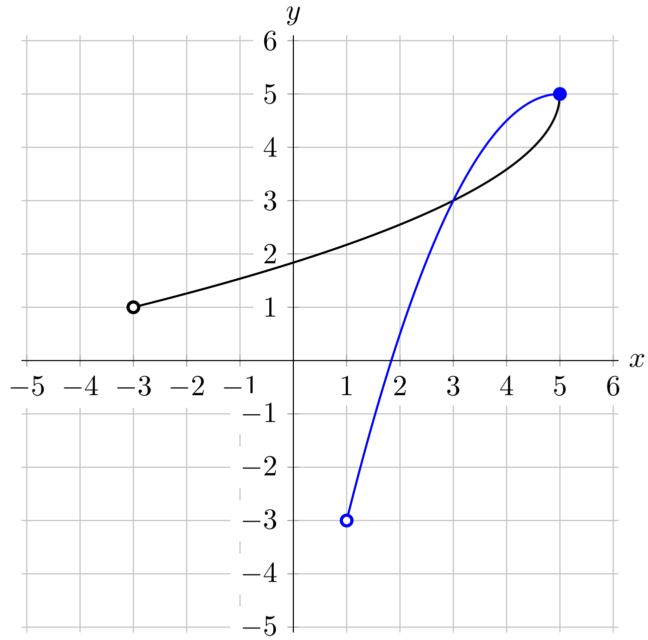
4. [5 points] Given below is the graph of a function $B(x)$. Briefly explain how you can tell that the function $B(x)$ is invertible. Then, on the same set of axes, carefully sketch the graph of $B^{-1}(x)$.

Explanation:

Solution: The function $B(x)$ is invertible because it passes the horizontal line test; or, because each output of $B(x)$ corresponds with only one input x.



5. [0 points] Suppose that $a(x) = mx + b$ for some constants m and b, where both m and b are not equal to zero.

In parts **a.** and **b.**, decide whether each of the following statements must be true, or whether it could be false, and circle the appropriate answer. You do not need to show work but limited partial credit may be available for work shown.

a. [2 points] If $f(x) = 3x - 5$, then the function $f(x) + a(x)$ must be linear.

[TRUE] FALSE

Solution: Note that $f(x) + a(x) = 3x - 5 + mx + b = (3 + m)x + (b - 5)$, which is also linear since m and b are constants.

b. [2 points] If $f(x) = 3x - 5$, then the function $f(x) \cdot a(x)$ must be linear.

TRUE [FALSE]

Solution: Now $f(x) \cdot a(x) = (3x - 5)(mx + b)$, which is quadratic since it will have a $3mx^2$ term if expanded, and $m \neq 0$.

Also define the function $q(x) = x^2 + 3$.

c. [3 points] If $a(q(x)) = \frac{1}{3}x^2$, find the values of m and b. **Show all work.**

Do not use these values of m and b for the other parts of this problem.

Solution:

Using the given formulas for a and q, we first find the composition $a(q(x)) = a(x^2 + 3) = m(x^2 + 3) + b = mx^2 + 3m + b$. Since we're told this must equal $\frac{1}{3}x^2$, we must have that $m = 1/3$. Then, since $3m + b$ must be 0, we can solve $3(1/3) + b = 0$, or $b = -1$.

$m =$ 1/3

$b =$ −1