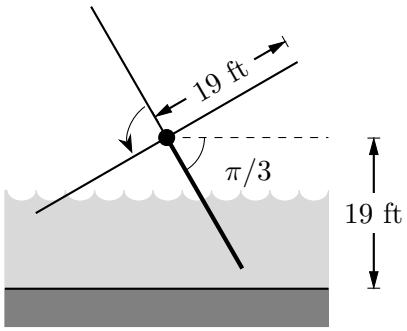
**6**. [0 points] You are standing by a river, watching two water wheels, each of which is rotating counterclockwise at a different but constant speed.

The first water wheel takes 24 seconds to complete a full revolution. Each blade of the wheel is 19 feet long, and when each blade is at its lowest point, it just barely scrapes the bottom of the river. One of the blades is painted red, shown as the thicker blade in the diagram to the right.




At the moment you begin watching, the red blade is exactly $\frac{\pi}{3}$ radians below the horizontal, as depicted. Let $r(t)$ be the height, in feet, of the tip of the red blade above the bottom of the river $t$ seconds after you begin watching.

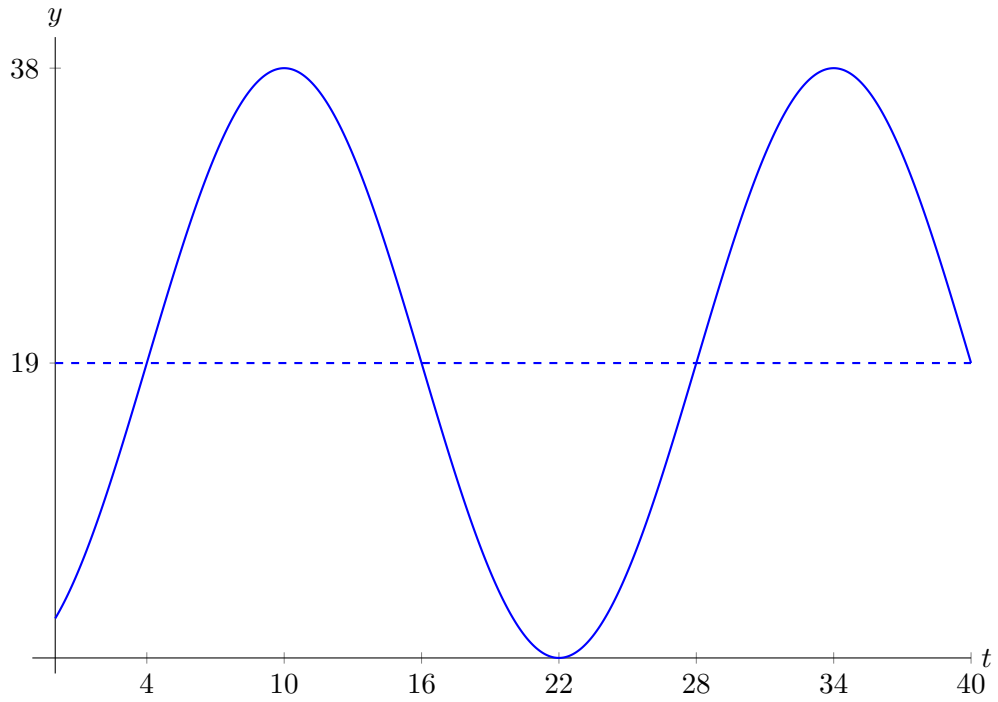
**a**. [1 point] How many seconds does it take the red blade to reach the horizontal position?

*Solution:* Since the blade starts $\pi/3$ radians below the horizontal and is moving counterclockwise, it needs to move 1/6 of a full rotation to reach the horizontal position. Since a full rotation is 24 sections, this will take $24/6 = 4$ seconds.

**Answer:** 4

**b**. [4 points] Sketch a graph of $y = r(t)$ on the interval $0 \le t \le 40$. Be sure the scales on your axes are clear, and pay careful attention to the shape of your graph.

*Solution:* Given part a, the graph should start below the midline of $y = 19$ and reach the midline at $t = 4$. Note that he period of the graph is 24, and the amplitude is 19.




**c**. [4 points] Find a formula for $r(t)$.

**Answer:** $r(t) =$ $19\sin(2\pi/24(t-4))+19$ **or** $19\sin(2\pi t/24-\pi/3)+19$

*This problem continues onto the following page.*

*This problem continues from the previous page and is restated for your convenience.*

You are standing by a river, watching two water wheels, each of which is rotating counterclockwise at a different but constant speed.

**d**. [5 points] The second water wheel has a blade painted blue, and you have determined that the height, in feet, of the tip of this blade above the bottom of the river $t$ seconds after you began watching is given by

$$20 + 15\sin\left(\frac{\pi}{8}t\right).$$

Find the **first three** positive values $t$ for which the height of the blade is 30 feet. **Show your work**, and give your answers in **exact** form.

*Solution:* We need to find when the height is 30, so we set the given formula equal to 30 to find one solution:

$$20 + 15\sin\left(\frac{\pi}{8}t\right) = 30$$

$$15\sin\left(\frac{\pi}{8}t\right) = 10$$

$$\sin\left(\frac{\pi}{8}t\right) = \frac{2}{3}$$

$$\frac{\pi}{8}t = \arcsin\left(\frac{2}{3}\right)$$

$$t = \frac{8}{\pi}\arcsin\left(\frac{2}{3}\right).$$

This will give us the first positive solution since $2/3 > 0$. Then, by using a sketch of $\sin\left(\frac{\pi}{8}t\right)$ (or of the given formula for the height), which has a period of 16, we can see that the second positive solution can be found by taking 8 minus the first solution. Finally, the third positive solution will be one period to the right of the first solution.

**Answers:**

$$\frac{8}{\pi}\arcsin\left(\frac{2}{3}\right)$$

$$8 - \frac{8}{\pi}\arcsin\left(\frac{2}{3}\right)$$

$$16 + \frac{8}{\pi}\arcsin\left(\frac{2}{3}\right)$$