7. [6 points] Consider the piecewise-defined function $k(x)$ given below. A portion of the graph of $k(x)$ is also shown for reference.

$$k(x) = \begin{cases} 
3e^{x-1} + 5 & x < 1 \\
3\ln(x) + 8 & x \geq 1
\end{cases}.$$ 

Find a formula for $x = k^{-1}(y)$. Be sure to show your work.

**Solution:** We set the first piece equal to $y$ and solve for $x$:

$$3e^{x-1} + 5 = y$$

$$e^{x-1} = \frac{y - 5}{3}$$

$$x - 1 = \ln\left(\frac{y - 5}{3}\right)$$

$$x = \ln\left(\frac{y - 5}{3}\right) + 1.$$ 

To find where this piece of the inverse function will apply, we look at the output values for $k(x)$ for $x < 1$, which is $5 < y < 8$.

Now we set the second piece equal to $y$ and solve for $x$:

$$3\ln(x) + 8 = y$$

$$\ln(x) = \frac{y - 8}{3}$$

$$x = e^{(y-8)/3}.$$ 

To find where this piece of the inverse function will apply, we look at the output values for $k(x)$ for $x \geq 1$, which is $y \geq 8$.

**Answer:** $k^{-1}(y) = \begin{cases} 
\ln\left(\frac{y-5}{3}\right) + 1 & \text{for } 5 < y < 8 \\
e^{(y-8)/3} & \text{for } 8 \leq y
\end{cases}$