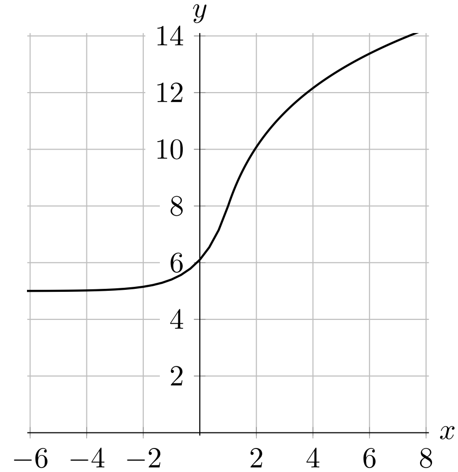
**7.** [6 points] Consider the piecewise-defined function $k(x)$ given below. A portion of the graph of $k(x)$ is also shown for reference.

$$k(x) = \begin{cases} 3e^{x-1} + 5 & x < 1 \\ 3\ln(x) + 8 & x \geq 1 \end{cases}.$$




Find a formula for $x = k^{-1}(y)$. Be sure to **show your work**.

*Solution:* We set the first piece equal to $y$ and solve for $x$:

$$3e^{x-1} + 5 = y$$
$$e^{x-1} = \frac{y-5}{3}$$
$$x - 1 = \ln\left(\frac{y-5}{3}\right)$$
$$x = \ln\left(\frac{y-5}{3}\right) + 1.$$

To find where this piece of the inverse function will apply, we look at the output values for $k(x)$ for $x < 1$, which is $5 < y < 8$.
Now we set the second piece equal to $y$ and solve for $x$:

$$3\ln(x) + 8 = y$$
$$\ln(x) = \frac{y-8}{3}$$
$$x = e^{(y-8)/3}.$$

To find where this piece of the inverse function will apply, we look at the output values for $k(x)$ for $x \geq 1$, which is $y \geq 8$.

**Answer:** $k^{-1}(y) = \begin{cases} \ln\left(\frac{y-5}{3}\right) + 1 & \text{for } 5 < y < 8 \\ e^{(y-8)/3} & \text{for } 8 \leq y \end{cases}$