1. [10 points] Below is a table giving some values of an **odd** function $f(x)$.
The domain of $f(x)$ is $(-\infty, \infty)$ (all real numbers).

x	2	3	4	5
$f(x)$	-3	-1	-1	1

a. [3 points] Find the following values of f, or write NEI if there is "not enough information" to find the value.

(i) $f(-2) =$ **3**

(ii) $f(1) =$ **NEI**

(iii) $f(0) =$ **0**

Solution:

(i) Because $f(x)$ is odd, we know that $f(-2) = -f(2)$. From the table, we know that this is 3.

(ii) The table doesn't tell us about $f(1)$ or $f(-1)$, so we don't have enough information to say what this value is.

(iii) Because $f(x)$ is odd, $f(0) = f(-0) = -f(0)$. The only way $f(0) = -f(0)$ is if $f(0) = 0$, so that is the value.

b. [2 points] Could f be an invertible function? Explain your answer.

The function f (*circle one*): COULD BE INVERTIBLE [COULD NOT BE INVERTIBLE]

Explanation:

Solution: $f(x)$ could not be invertible. Because $f(3) = f(4) = -1$. Because two different inputs produce the same output, the function is not invertible.

c. [4 points] Recall that $f(x)$ is an odd function. For each of the following functions, decide whether it is even, odd, neither, or if there is not enough information (NEI) to tell. *No explanation needed.*

(i) The function $g(x) = x^3 f(x)$ is (*circle all that apply*):

ODD [EVEN] NEITHER NEI

(ii) The function $h(x) = x^2 + f(x)$ is (*circle all that apply*):

ODD EVEN [NEITHER] NEI

Solution:

(i) First note that $(-x)^3 = -x^3$. Then since $f(x)$ is odd,

$$g(-x) = (-x)^3 f(-x) = (-x^3)(-f(x)) = x^3 f(x) = g(x)$$

Thus $g(x)$ is even.

(ii) We can test one of the values of the table along with its negative, and see that $h(x)$ is neither even nor odd. For example, $h(2) = 4 - 3 = 1$, while $h(-2) = 4 + 3 = 7$. Since 1 and 7 are not equal or negatives of each other, $h(x)$ is neither even nor odd.

d. [1 point] Suppose it is also true that: $\lim_{x\to\infty} f(x) = 5$. Use this information to find $\lim_{x\to-\infty} f(x)$, or write NEI if there is not enough information to find the limit.

Solution: We know that $f(x)$ approaches 5 when x gets very large. Because $f(x)$ is odd, as x gets very negative, it will approach the opposite value of -5.

$\lim_{x\to-\infty} f(x) =$ -5