3. [11 points] The Go Blue Zoo's power has gone out and it is cold outside! The indoor frog exhibit is typically kept warm, but it is now getting colder. The exhibit temperature, in °F, t hours after the power goes out is given by:

$$E(t) = 30 + we^{kt}, \quad \text{where } w \text{ and } k \text{ are constants.}$$

a. [3 points] When the power first went out, the temperature of the frog exhibit was $75°F$, but after 4 hours the temperature is 68°F. Find the values of w and k.
Show all work. Give your answers in exact form, or accurate to two decimal places.

Solution: The first fact tells us that $E(0) = 75$, so

$$75 = 30 + we^(k \cdot 0) = 30 + w$$

Solving for w shows us $w = 45$.
The second fact tells us that $E(4) = 68$, so

$$68 = 30 + we^(k \cdot 1) = 30 + we^k = 30 + 45e^k$$

Solving for e^k shows us $e^k = 38/45$. Then, taking the natural logarithm of both sides, we get

$$k = ln(38/45).$$

$w =$ ________________ $k =$ ________________

b. [2 points] What is $\lim_{t\to\infty} E(t)$? Explain what this number means in the context of the problem.

Solution: Because $32/38 < 1$, $ln(32/38) < 0$, and the exponential expression $e^(ln(32/38)t)$ goes to 0 as $t \to \infty$. Thus $B(t)$ goes to 30 as $t \to \infty$. This is the temperature the frog exhibit will approach in the long run, which is the temperature of the environment outside.

$\lim_{t\to\infty} E(t) =$ __________

Meaning:

c. [4 points] Last summer the Go Blue Zoo also had a power outage. This time it was hot outside and the refrigerator for storing the tiger's food started warming up. After t hours, the temperature inside the refrigerator, in $°F$, was given by

$$R(t) = 75 - 38e^{-0.03t}$$

Due to safety concerns, food must be thrown out if the temperature inside a refrigerator rises above 40°F. How long could the power outage last without having to throw out the tiger's food? *Show all work. Give your answer in exact form, or accurate to two decimal places.*

Solution: We want to find the value of t such that

$$R(t) = 75 - 38e^{-0.03t} = 40$$

Isolating the exponential term, we get

$$35/38 = e^{-0.03t}$$

Then, taking the natural logarithm of both sides, we get

$$\ln(35/38) = -0.03t$$

so $t = \frac{\ln(35/38)}{-0.03}$.

______________________ hours

This problem continues on the next page.

d. [2 points] Recall that we're considering the function

$$R(t) = 75 - 38e^{-0.03t}$$

which tracks the temperature inside a refrigerator, in $^\circ F$, t hours after the power goes out on a hot day.

The domain of that function in *this* context would be $t \geq 0$; however, to help you make sense of meaningful features, the graphs below are shown on a larger domain.

Which of the four graphs show below could be the graph of $R(t)$? (*Circle one*)