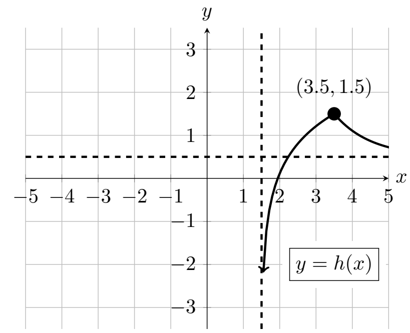
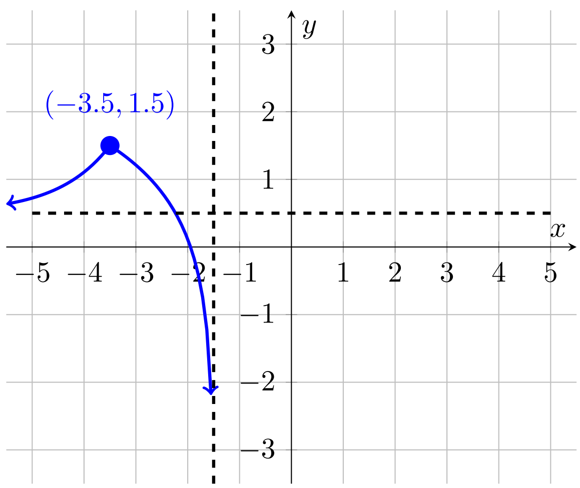
4. [11 points]

a. [6 points]

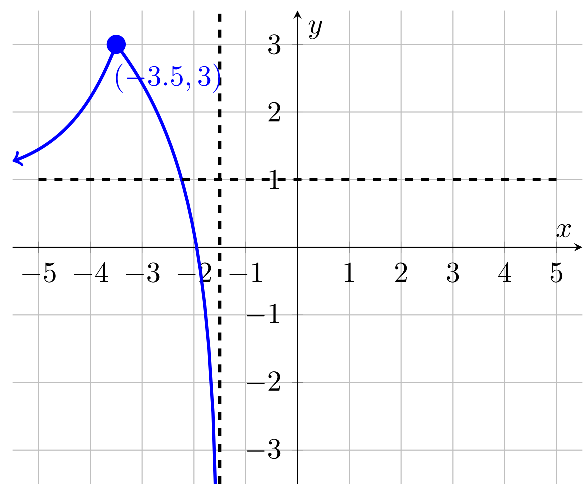
To the right is a graph of a function $h(x)$. The graph of $h(x)$ goes through the corner point $(3.5, 1.5)$, has a horizontal asymptote at $y = 0.5$, and a vertical asymptote at $x = 1.5$. We will apply a sequence of transformations to the graph of $h(x)$. For each subsequent transformation, sketch the intermediate graphical result on the given set of axes. **For each step, clearly denote any asymptotes with a dotted line, and label coordinates of the graph's corner point.**



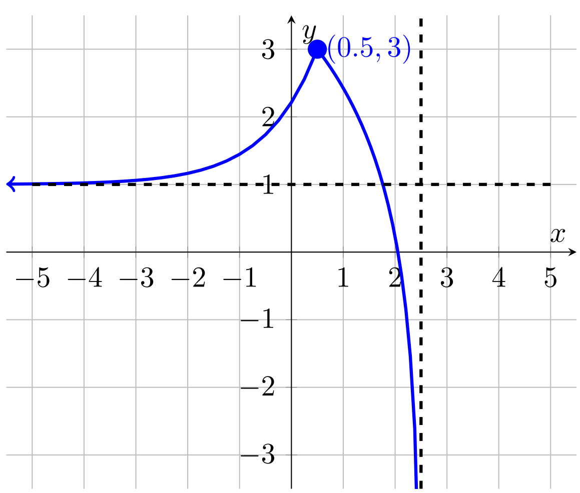
Step 1: Reflect the graph of $h(x)$ over the y-axis.



Step 2: Stretch the graph from Step 1 vertically by a factor of 2.



Step 3: Shift the graph from Step 2 right by 4.



This problem continues on the next page.

b. [3 points] Consider the function represented by the final graph in Step 3 and call it $g(x)$. Give a formula for $g(x)$ in terms of the original function $h(x)$.

$g(x) =$ $2h(-(x-4))$

c. [2 points] Use the final coordinates of your corner point in the graph of $g(x)$ (that is, Step 3) to check if your formula in part (b) is correct. *You can get points for this part of the problem even if your formula above is incorrect and you figure that out in this step, but don't know how to correct your formula.*

Solution:
Let's check that $g(0.5) = 3$, as is indicated by the corner point in our final graph. We can do that by plugging in 0.5 and going from there:

$$\begin{aligned} g(0.5) &= 2h(-(0.5-4)), \text{ using our proposed formula from part (b)} \\ &= 2h(-(-3.5)) \\ &= 2h(3.5) \\ &= 2(1.5), \text{ since our original corner point told us that } h(3.5) = 1.5 \\ &= 3 \end{aligned}$$

By plugging in 0.5 we see that our output is 3, which is also what we got in our final graph. This is good evidence that our formula is, indeed, correct!