6. [7 points] The Go Blue Zoo breeds its own insect populations for feeding some of the birds and reptiles. However, a virus infected the cricket breeding program on January 1, 2023. The cricket population started at 1,000,000 crickets, but decreased by 15% per week once the colony was infected.

   a. [3 points] How many weeks did it take for the population to decrease by 60%? Show all work. Give your answer in exact form, or accurate to at least two decimal places.

   Solution:

   We need to solve the following equation for $t$:

   $$400,000 = 1,000,000(0.85)^t$$

   This simplifies to:

   $$0.4 = 0.85^t$$

   Taking log of both sides we get

   $$\log(0.4) = \log(0.85^t)$$

   And then applying log rules we get:

   $$\log(0.4) = t \log(0.85)$$

   Finally, isolating $t$ using division we get:

   $$\frac{\log(0.4)}{\log(0.85)} = t$$

   weeks

   b. [2 points] By what percentage did the cricket population decrease for each day? Show all work. Give your answer in exact form, or accurate to at least two decimal places.

   Solution: One way to think about this problem is to think about solving for an unknown daily decay factor $d$ that satisfies $d^7 = 0.85$. That is, a daily decay factor, when applied seven days in a row, should be equivalent to 0.85. If we solve for $d$ we get

   $$d = 0.85^{\frac{1}{7}}$$

   But that tells us the daily decay factor. To find out the percent decrease per day we need to compute $100 \times \left(1 - 0.85^{\frac{1}{7}}\right)$.

   Note that it’s tempting to use the known formula $r = b - 1$. That would, correctly, give us $100 \times \left(0.85^{\frac{1}{7}} - 1\right)$ as a percent change. However, that would give us the rate as a negative percent (because it’s decaying). That’s already captured in the fact that we’re using the word “decay”, so we want to make sure to give that as a positive number. That is, we wouldn’t say something “decays by $-5\%$ daily”.

   Decreases by $100 \times \left(1 - 0.85^{\frac{1}{7}}\right)$ % per day.
c. [2 points] If the population instead decayed at a \textit{continuous} rate of 15\% per week, by what \textit{non-continuous} percentage would it decrease in one week? \textit{Show all work. Give your answer in exact form, or accurate to at least two decimal places.}

\textbf{Solution:} Similar to part (b), we need to find a decay factor, and then convert that to a percent change. In this case, if our continuous decay rate is 15\%, then our growth (or decay) factor is \( b = e^{-0.15} \). So our decay rate is

\[100 \times (1 - e^{-0.15})\]

Decreases by \( \frac{100 \times (1 - e^{-0.15})}{(1 - e^{-0.15})} \) \% per week.