6. [7 points] The Go Blue Zoo breeds its own insect populations for feeding some of the birds and reptiles. However, a virus infected the cricket breeding program on January 1, 2023. The cricket population started at 1,000,000 crickets, but decreased by 15% per week once the colony was infected.

a. [3 points] How many weeks did it take for the population to decrease by 60%? *Show all work. Give your answer in exact form, or accurate to at least two decimal places.*

Solution:
We need to solve the following equation for t:

$$400,000 = 1,000,000(0.85)^t$$

This simplifies to:

$$0.4 = 0.85^t$$

Taking log of both sides we get

$$\log(0.4) = \log(0.85^t)$$

And then applying log rules we get:

$$\log(0.4) = t\log(0.85)$$

Finally, isolating t using division we get:

$$\frac{\log(0.4)}{\log(0.85)} = t$$

$\frac{\log(0.4)}{\log(0.85)}$ weeks

b. [2 points] By what percentage did the cricket population decrease for each *day*? *Show all work. Give your answer in exact form, or accurate to at least two decimal places.*

Solution: One way to think about this problem is to think about solving for an unknown daily decay factor d that satisfies $d^7 = 0.85$. That is, a daily decay factor, when applied seven days in a row, should be equivalent to 0.85. If we solve for d we get

$$d = 0.85^{\frac{1}{7}}$$

But that tells us the daily decay *factor*. To find out the percent decease per day we need to compute $100 \times \left(1 - 0.85^{\frac{1}{7}}\right)$.
Note that it's tempting to use the known formula $r = b - 1$. That would, correctly, give us $100 \times \left(0.85^{\frac{1}{7}} - 1\right)$ as a precent *change*. However, that would give us the rate as a *negative* percent (because it's decaying). That's already captured in the fact that we're using the word "decay", so we want to make sure to give that as a positive number. That is, we wouldn't say something "decays by −5% daily".

Decreases by $100 \times \left(1 - 0.85^{\frac{1}{7}}\right)$ % per day.

c. [2 points] If the population instead decayed at a *continuous* rate of 15% per week, by what *non-continuous* percentage would it decrease in one week? *Show all work. Give your answer in exact form, or accurate to at least two decimal places.*

Solution: Similar to part (b), we need to find a decay factor, and then convert that to a percent change. In this case, if our continuous decay rate is 15%, then our growth (or decay) *factor* is $b = e^{-0.15}$. So our decay *rate* is

$$100 \times (1 - e^{-0.15})$$

Decreases by $100 \times (1 - e^{-0.15})$ % per week.