

1. [12 points] Below is a graph of periodic, odd function h(t), with period 6:

Solution: Because g(t) is a horizontal compression of h(t) by a factor of 1/2, its period will be half as large, so 3.

When we horizontally stretch h(t) and reflect it over the *t*-axis, it still retains its symmetry about the origin, so is still odd. We could also see this algebraically, using the fact that h(t) is odd:

$$g(-t) = -h(-2t) = -(-h(2t)) = -g(t)$$

The amplitude of h(t) is unchanged by the transformations, so the amplitude of g(t) is still 3.

b. [3 points] Another **new** function $w(t) = 3h(t-1) \dots$

- ... has period: <u>6</u>
- ... is (CIRCLE ONE) ODD EVEN NEITHER
- ... has maximum value: _____9

Solution: Because w(t) has not been stretched or compressed horizontally, its period will remain the same: 6.

When we horizontally shift h(t) right by 1, it loses its symmetry about the origin. It is neither odd nor even after that shift.

w(t) is a vertical stretch of h(t) by a factor of 3, so its amplitude is 3 times as large, so 9.



c. [6 points] Carry out the following sequence of transformations to the graph of h(t). Draw each intermediate graph on the provided axes. Clearly label at least three specific, known points in each graph.

1. Shift the graph of h(t) up by 1 unit.



2. Reflect the resulting graph over the *t*-axis.



Call the function in the final graph k(t). What is a formula for k(t) in terms of h(t)?

k(t) = -(h(t) + 1) or, equivalently, -h(t) - 1