2. [9 points] A group of scientists is modeling the transmission of light through different liquids. The functions below measure the brightness of the light, in lumens, at a depth of d cm below the surface of two different liquids: A and B.

$$A(d) = 45e^{-0.001d}$$
$$B(d) = 50e^{-0.001(2d-25)}$$

The functions $A(d)$ and $B(d)$ have a domain of $[0, \infty)$.

a. [1 point] How bright is the light at the surface of liquid B? *Express your answer in exact form, or rounded to at least two decimal places.*

Solution:
The brightness at the surface of liquid B is $B(0) = 50e^{-0.001(0-25)} = 50e^{0.025} \approx 51.27$ lumens.

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b. [4 points] At what depth do the lights in the experiments with liquids A and B have the same brightness? *Show all work. Express your answer in exact form, or rounded to at least two decimal places.*

Solution:
To find where the brightnesses are equal, we need to solve $A(d) = B(d)$.

$$45e^{-0.001d} = 50e^{-0.001(2d-25)} \implies \ln(45) - 0.001d = \ln(50) - 0.001(2d-25)$$
$$\implies 0.001d = \ln(50) - \ln(45) + 0.025$$
$$d = \frac{\ln(50) - \ln(45) + 0.25}{0.001} \text{ cm}$$
$$d \approx 130.36 \text{ cm.}$$

$\frac{\ln(50) - \ln(45) + 0.25}{0.001} \approx 130.36$ cm

c. [4 points] In a third experiment the scientists observe that the brightness of a light decreases by 10% for every 5 cm of depth below the surface of a liquid C. No matter the starting depth, how much deeper do you need to go to reduce the brightness by 25%? *Show all work. Express your answer in exact form, or rounded to at least two decimal places.*

Solution:
$(0.9)^{d/5} = 0.75 \implies \frac{d}{5}\log(0.9) = \log(0.75) \implies d = \frac{5\log(0.75)}{\log(0.9)} \approx 13.65$ cm.

$\frac{5\log(0.75)}{\log(0.9)} \approx 13.65$ cm