3. [11 points] Asteroid Mining Co. hauls mineral-rich asteroids from across the solar system back to Earth's orbit for mining. The scientists at the company use the following functions to compute instructions for the space ship's crew.

- $S(p)$ is the amount of fuel, measured in liters, that is needed to move the ship p parsecs (a unit of distance).
- $F(s)$ is the amount of fuel, measured in liters, that is used when the engines run for s seconds.

Assume that $S(p)$ and $F(s)$ are both invertible.

a. [6 points] For each of the expressions below, give an interpretation or explain why the expression doesn't make sense.

- $S^{-1}(2.3 \times 10^8) = 0.2$

Solution:
The space ship needs 2.3×10^8 liters of fuel to travel 0.2 parsecs.

- $S(F(60))$

Solution:
This composition doesn't make sense. The output of F is liters of fuel, but the input of S is distance in parsecs.

- $F^{-1}(S(20))$

Solution:
The amount of time, in seconds, that the engines need to run for the ship to move 20 parsecs.

b. [2 points] Write an expression for the amount of fuel used, in liters, when the ship travels 2.8 parsecs and then runs the engines for an additional 30 seconds. Your answer may include any of S, F, S^{-1}, or F^{-1}.

$S(2.8) + F(30)$ liters

c. [3 points] Define the function $D(m)$ to be the distance, in parsecs, the ship moves when the engines run for m **minutes**. Write a formula for $D(m)$ in terms of F, S, F^{-1}, and/or S^{-1}.

Solution: The composition: time (seconds) → fuel (liters) → distance (parsecs) is $S^{-1}(F(s))$, where s is time measured in seconds. A minute is 60 seconds, so $s = 60m$.

$D(m) =$ $S^{-1}(F(60m))$