4. [8 points] The following parts are unrelated.

a. [4 points] The quantity of an intravenous drug in a patient's body, in mg, is given by $D(t) = 250(0.88)^t$, where t is the number of hours after the drug was administered.

What is the hourly decay rate of the drug? *Express in exact form, or rounded to at least 2 decimal places.*

Solution: We can see in the formula that the growth factor, b, is 0.88. This means that 88% of our drug remains after each hour, so it has decayed by 12%. This means our decay rate is 12%. We could also find this using $100 \times (1 - 0.88)$.

12%

What is the ***continuous*** hourly decay rate of the drug? *Express in exact form, or rounded to at least 2 decimal places.*

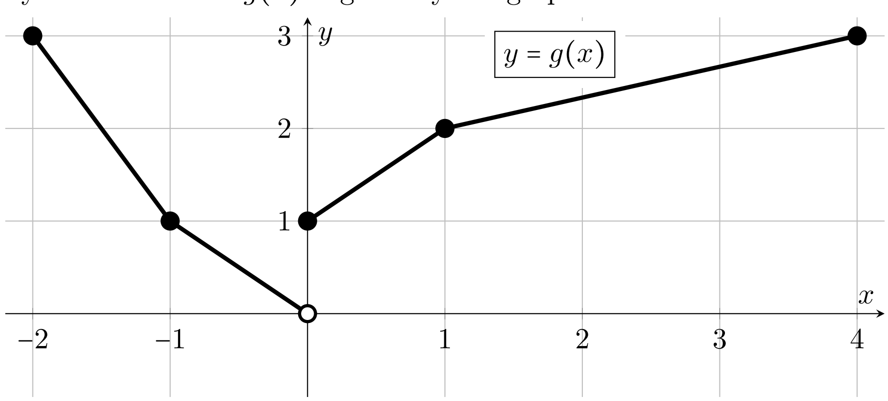
Solution: $e^k = 0.88$, so $k = \ln(0.88) \approx -0.12783$. This means that the continuous decay rate is approximately 12.783%.

12.783%

b. [4 points] The function $f(x)$ is given by the following formula:

$$f(x) = \ln(x) + 2$$

The entirety of the function $g(x)$ is given by the graph below.



Find the following values, or write NEI if there is "not enough information" to compute them. *Show all work.*

- $f^{-1}(g(0)) = \underline{\quad \frac{1}{e} \quad}$

Solution: First, we can use the graph to find that $g(0) = 1$. Thus we need to compute $f^{-1}(1)$. One way to do this is to solve the following for x:

$$\ln(x) + 2 = 1$$

That is, finding the value of x that gave an output of 1. We can solve that as follows:

$$\ln(x) = -1$$

$$e^{-1} = \frac{1}{e} = x$$

We could have also found a general formula for f^{-1} and evaluated it at 1. That general formula would turn out to be:

$$f^{-1}(y) = e^{y-2}$$

giving the same value of $f^{-1}(1) = e^{1-2} = e^{-1}$ as above.

- All x such that $g(g(x)) = 2$: $x =$ $\underline{\quad -1, 0 \quad}$

Solution: We want to know where the outer function $g(\ldots)$ gives an output of 2. It gives an output of 2 exactly when its input is 1 or when its input is -1.5. That is, we need to know when the inner function (also $g(x)$ in this case!) will give us either 1 or -1.5. The number -1.5 is not in the range of $g(x)$, so we only need to solve for when $g(x) = 1$. This happens exactly when $x = -1, 0$. Let's plug those numbers back in to check that we get what we were hoping for:

$$g(g(-1)) = g(1) = 2 \checkmark$$
$$g(g(0)) = g(1) = 2 \checkmark$$