5. [10 points] The temperature T in a given room, measured in °F, after an air conditioner is turned on is given by $T = f(t) = 68 + 5e^{-0.02t}$, where t is measured in minutes.

a. [4 points] Find the following limits of $f(t)$:

(i) $\lim_{t\to\infty} f(t) =$ **68**

(ii) $\lim_{t\to-\infty} f(t) =$ ∞

b. [3 points] Find a formula for $t = f^{-1}(T)$.

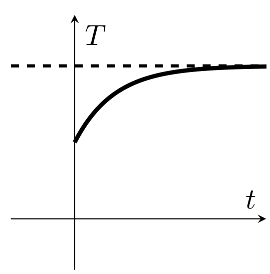
Solution: Starting with $T = 68+5e^{-0.02t}$, our goal is to isolate t, thereby finding an expression for t as a function of T.

$$\begin{aligned} T &= 68 + 5e^{-0.02t} \\ T - 68 &= 5e^{-0.02t} \\ \frac{T-68}{5} &= e^{-0.02t} \\ \ln\left(\frac{T-68}{5}\right) &= -0.02t \\ -\frac{1}{0.02}\ln\left(\frac{T-68}{5}\right) &= -50\ln\left(\frac{T-68}{5}\right) = t \end{aligned}$$

$f^{-1}(T) =$ $-50\ln\left(\frac{T-68}{5}\right)$

c. [3 points]

The graph of $T = q(t)$ to the right shows the temperature in a different room when being *heated* as a function of time t. The domain shown is $[0, \infty)$ and the dashed line represents a horizontal asymptote of $q(t)$.



Given that behavior, which of the following *could* be a formula for $q(t)$? (*Circle all that apply.*)

$q(t) = 3\log(t+2)$

$\boxed{q(t) = -0.7^t + 65}$

$\boxed{q(t) = -e^{-0.2t} + 67}$

$q(t) = 50 \cdot 1.02^t$

$q(t) = -3\log(t+5)$

$q(t) = -e^{0.1t} + 69$

Solution: As this graph has a horizontal asymptote, it cannot be either of the functions involving log, which only have vertical asymptotes. Further, it cannot be $q(t) = 50 \cdot 1.02^t$, because this is simply an exponentially increasing function and should only have a horizontal asymptote at $T = 0$.
Of the remaining options, $q(t) = -0.7^t + 65$ and $q(t) = -e^{-0.2t} + 67$ are both exponentially decreasing functions that have been reflected over the t-axis and then shifted up: exactly the behavior we see in the desired graph. The last option $q(t) = -e^{0.1t} + 69$ is an exponentially *increasing* function that has reflected over the t-axis and then shifted up, which is not what we see in our desired graph.