5. [9 points] Matthew bakes chocolate soufflés to sell in his restaurant, and he is testing how the soufflés cool so that he can serve them at the perfect temperature.

a. [4 points] In his home kitchen, the temperature of a soufflé, in degrees Celsius (°C), after being out of the oven for t seconds, is given by

$$H(t) = 177e^{kt} + c, \qquad \text{where } c \text{ and } k \text{ are constants.}$$

Matthew finds that the temperature of a soufflé at the moment it comes out of the oven is 195°C. After 100 seconds, it has cooled to 60°C. Find the values of c and k. *Show all your work. Give your answers in exact form or rounded to 3 decimal places.*

Solution: Since

$$H(0) = 177e^{k\cdot 0} + c = 177 + c = 195,$$

we must have that $c = 18$. Then,

$$H(100) = 177e^{k\cdot 100} + 18 = 60$$

$$e^{k\cdot 100} = \frac{42}{177}$$

$$k = \frac{1}{100}\ln\left(\frac{42}{177}\right)$$

$c =$ 18 $\qquad k =$ ≈ -0.014

b. [3 points] When he moves to his restaurant kitchen, the temperature of a soufflé, in °C, after being out of the oven for t seconds is instead given by

$$R(t) = 188e^{-0.01t} + 22.$$

After taking a soufflé out of his restaurant's oven, how long should Matthew wait to serve it if he wants it to be 80°C at that moment? *Show all your work. Give your answers in exact form or rounded to 3 decimal places.*

Solution: We set $R(t) = 80$ and solve for t:

$$188e^{-0.01t} + 22 = 80$$

$$e^{-0.01t} = \frac{58}{188}$$

$$t = \frac{\ln\left(\frac{58}{188}\right)}{-0.01}$$

117.600 seconds

c. [2 points] Find $\lim_{t\to\infty} R(t)$, then interpret what it means in the context of this problem, including any relevant units.

$\lim_{t\to\infty} R(t) =$ 22

Interpretation:

Solution: If Matthew let the soufflé cool for a very long time, its temperature would approach 22°C.