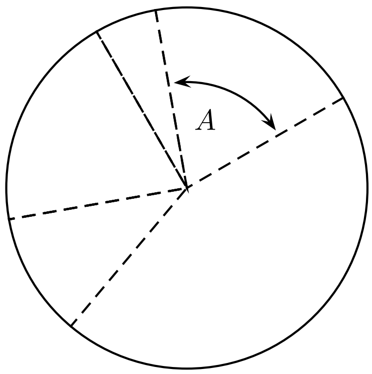
10. [9 points] There is a pumpkin pie in the shape of a circle of radius 12 centimeters. The pie is sliced by making cuts along radii, as pictured below. (Slices are NOT necessarily the same size.) *Show your work. All answers should be in exact form or be accurate to at least 3 decimal places.*



Note: Figure (and slices) not drawn to scale.

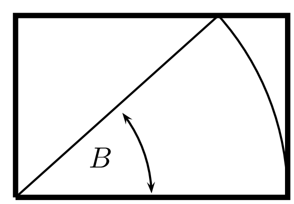
a. [2 points] The pie is surrounded by a thin crust. The first slice of pie you take has angle measuring A radians. (See picture.) If the length of crust on your slice is 9 centimeters, compute the value of A.

Solution: Using the arclength formula, we have $9 = 12A$, so $A = 9/12 = 0.75$.

$A =$ 0.75

b. [3 points] You have a plate in the shape of a rectangle of length 12 cm and width 8 cm. Your second slice of pie has angle measuring B, in radians. Find the maximum value of B so that your slice of pie will fit on the plate (as shown in the picture below).

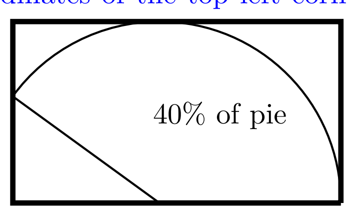
Solution: If we think of the center of the pie as the origin $(0,0)$, then the coordinates of the top right corner of the piece of pie are $(12\cos B, 12\sin B)$. So, the largest possible value of B satisfies $12\sin B = 8$. That is, $\sin B = 8/12$ and $0 \le B \le \pi/2$, so $B = \arcsin(8/12) \approx 0.7297$.



Answer: $\arcsin(2/3) \approx 0.7297$

c. [4 points] At the end of the evening, after helping yourself to several slices, 40% of the pie remains. The pie will be placed in a rectangular tupperware container (as shown in the picture below) to be refrigerated. Find the length, ℓ, and width, w, both measured in centimeters, of the smallest tupperware container into which the remaining pie will fit.

Solution: First note that $w = 12$ since at least 25% of the pie remains. Now, the angle measurement of the remaining 40% of the pie is 40% of 2π which is $0.4(2\pi) = 0.8\pi$. If we let the center of the pie be the origin $(0,0)$, then the coordinates of the top left corner of the remaining pie shown are $(12\cos(0.8\pi), 12\sin(0.8\pi))$. Hence the coordinates of the lower corners of the rectangular container are $(12\cos(0.8\pi), 0)$ and $(12, 0)$. Thus $\ell = 12 - 12\cos(0.8\pi) \approx 21.708$.



$w =$ 12

$\ell =$ $12 - 12\cos(0.8\pi) \approx 21.708$