2. [10 points] A movie theater is considering selling discount tickets for opening night of a new vampire movie. The management estimates that they will sell 1100 tickets if they set the price of tickets at \$7 each. However, if they charge \$10 for each ticket, the theater will only sell 800 tickets. Let $T(p)$ be the number of tickets the theater will sell if the price of each ticket is p dollars. Assume that $T(p)$ is a linear function.

a. [4 points] Find a formula for $T(p)$ in terms of p.

Solution: The slope (average rate of change) is

$$\frac{T(10)-T(7)}{10-7} = \frac{800-1100}{10-7} = -100$$

tickets per dollar. Using point-slope form, we find $T(p) = 800-100(p-10) = 1800-100p$.

$T(p) =$ $1800 - 100p$

b. [1 point] Let $R(p)$ be the total amount of money the theater takes in from ticket sales if the price of each ticket is p dollars. Find a formula for $R(p)$ in terms of p.

Solution: If $T(p)$ tickets are sold at a price of p dollars each, then the total amount of money taken in from ticket sales is $R(p) = p(T(p)) = p(1800-100p) = 1800p - 100p^2$.

$R(p) =$ $1800p - 100p^2$

c. [5 points] By completing the square, put $R(p)$ in vertex form. *Show step by step work.* How much should the theater charge for each ticket if they want to maximize the amount of money they take in? How much would the theater take in if they charged this amount?

Solution: Using the method of completing the square, we find

$$\begin{aligned} R(p) &= -100p^2 + 1800p = -100(p^2 - 18p) \\ &= -100(p^2 - 18p + 81 - 81) \\ &= -100\left((p-9)^2 - 81\right) = -100(p-9)^2 + 8100. \end{aligned}$$

Hence the vertex of the graph of $R(p)$ is $(9, 8100)$. Since the leading coefficient is negative, the parabola opens downward and the vertex gives a maximum. Therefore, the maximum of $R(p)$ is 8100 and occurs at $p = 9$, i.e. the maximum revenue of the theater would be \$8100 at a ticket price of \$9.

Vertex form: $R(p) =$ $-100(p-9)^2 + 8100$

Ticket price: \$9 **Money taken in:** \$8100