5. [10 points]

a. [2 points] Consider the function $P(t)$ defined by

$$P(t) = \begin{cases} \frac{70t(t-6)}{(t-10)(t+2)} & \text{if } 0 \le t \le 5 \\ 2 + 5e^{5-t} & \text{if } t > 5. \end{cases}$$

Evaluate $P(5)$ and $P(P(5))$.

Solution: Since $5 \le 5$, we use the first part of the formula to find $P(5)$. So $P(5) = \frac{70(5)(5-6)}{(5-10)(5+2)} = 10$. Then $P(P(5)) = P(10) = 2 + 5e^{5-10} = 2 + 5e^{-5}$. (Note that we use the second part of the formula for $P(t)$ to compute $P(10)$ since $10 > 5$.)

$P(5) =$ 10 $\qquad P(P(5)) =$ $2 + 5e^{-5}$

b. [4 points] Below, you are given a table with some data about two functions: $f(t)$ and $h(t)$. You are also given information about some transformations and combinations of these functions. Fill in the missing entries in the table. You may assume $f(t)$ and $h(t)$ are invertible functions. *No work or explanation is required.*

t	0	1	2	3
$f(t)$	2	4	5	9
$h(t)$	3	8	1	7
$f(h(t))$	9	6	4	11
$f^{-1}(t)$	12	11	0	10
$f(t+3)$	9	7	8	12

c. [4 points] Suppose $g(x)$ is a power function such that $g(1) = 3$ and $g(5) = 6$. Find a formula for $g(x)$ in terms of x. *Give your answer in exact form.*

Solution: Since $g(x)$ is a power function, it can be written in the form $g(x) = kx^p$ for some constants k and p.
Using the given data, we have $g(1) = 3$, so $3 = k(1^p)$ and thus $3 = k$. To find p, we use the fact that $g(5) = 6$ and find $6 = 3(5^p)$, so $2 = 5^p$. Taking the natural log of both sides of this equation, we see that $\ln 2 = p \ln 5$ so $\ln 2/\ln 5 = p$. Thus a formula for $g(x)$ is $g(x) = 3x^{\ln 2/\ln 5}$.

$g(x) =$ $3x^{\ln 2/\ln 5}$