6. [12 points] *Remember to show your work carefully. All numbers appearing in your answers should either be in* exact form *or be accurate to at least 3 decimal places.*

Authorities in Volterra, Italy noticed an increase in the sales of extra-strength dental floss at supermarkets in the city during the early part of the year 2010. Let $D = p(t)$ denote the quantity of extra-strength dental floss, in meters, sold in Volterra on day t of 2010 (where $t = 1$ represents January 1). We are told that $p(t)$ is an exponential function.

a. [5 points] 400 meters of extra-strength dental floss were sold in Volterra on January 7, and 600 meters of extra-strength dental floss were sold on January 23. Find a formula for $p(t)$ in terms of t.

Solution: Since $p(t)$ is exponential, there are constants a and b so that $p(t) = ab^t$. The information provided tells us that $p(7) = 400$ and $p(23) = 600$, so $400 = ab^7$ and $600 = ab^{23}$. Dividing, we see that $\frac{600}{400} = \frac{ab^{23}}{ab^7}$ so $1.5 = b^{16}$. Thus $b = \sqrt[16]{1.5} \approx 1.026$.
To find a, we use the value of b we just found and the equation $400 = ab^7$ to see that $400 = a(1.5)^{7/16}$ so $a = 400(1.5^{-7/16}) \approx 334.981$. Hence a formula for $p(t)$ is $p(t) = 400(1.5^{-7/16})(\sqrt[16]{1.5})^t = 400(1.5^{(t-7)/16}) \approx 334.981(1.026)^t$.

$p(t) =$ $400(1.5^{-7/16})(\sqrt[16]{1.5})^t = 400(1.5^{(t-7)/16}) \approx 334.981(1.026)^t$

b. [3 points] How long does it take for the quantity of extra-strength dental floss sold each day to double?

Solution: With a and b as in part (a), we want to find t so that $p(t) = 2p(0)$, i.e. so that $ab^t = 2a$ or $b^t = 2$. Using the natural logarithm, we see that $t \ln b = \ln 2$ so $t = \ln 2 / \ln b = \ln 2 / \ln(\sqrt[16]{1.5}) = 16 \ln 2 / \ln 1.5 \approx 27.352$. Hence the quantity of extra-strength dental floss sold each day doubles in $16 \ln 2 / \ln 1.5$ (just over 27) days.

Answer: $\frac{16 \ln 2}{\ln 1.5}$ **days**

c. [4 points] Sales of flea shampoo have also been increasing. If $F = q(t)$ is the quantity of flea shampoo, in grams, sold in Volterra on day t of 2010, then $q(t) = \ln(t + 1) + 65$. Find a formula for $q^{-1}(F)$ in terms of F.

Solution: We solve for t in the formula $F = \ln(t + 1) + 65$ and find

$$\begin{aligned} F &= \ln(t+1) + 65 \\ F - 65 &= \ln(t+1) \\ e^{F-65} &= t + 1 \\ e^{F-65} - 1 &= t \end{aligned}$$

So $q^{-1}(F) = e^{F-65} - 1$.

$q^{-1}(F) =$ $e^{F-65} - 1$