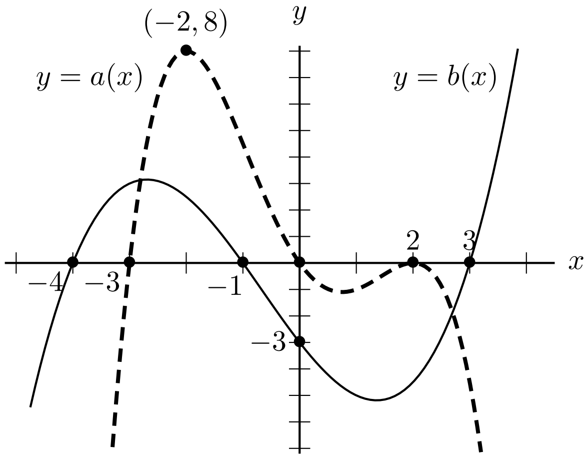
8. [12 points] The graphs of two polynomials $a(x)$ (dashed line) and $b(x)$ (solid line) are shown below. Assume all the key features of the graphs are shown. Note: No work or explanation is required for parts (a)–(c). However, partial credit *may* be awarded for work shown.



a. [2 points] Evaluate $b(a(2))$.

Solution: From the graph, we see that $b(a(2)) = b(0) = -3$.

Answer: $b(a(2)) = -3$

b. [4 points] Find the zero(s) and vertical asymptote(s) of the function $\dfrac{a(x)}{b(x)}$.

Solution: Note that $a(x)$ and $b(x)$ have no common zeros.
The zeros of $\dfrac{a(x)}{b(x)}$ are the zeros of the numerator $a(x)$, i.e. $x = -3, 0$, and 2.
The vertical asymptotes of $\dfrac{a(x)}{b(x)}$ are given by the zeros of the denominator $b(x)$, so the vertical asymptotes are $x = -4$, $x = -1$, and $x = 3$.

Zero(s): $x = -3, 0, 2$ **Vertical asymptote(s)**: $x = -4$, $x = -1$, $x = 3$

c. [2 points] Estimate the horizontal intercept(s) of the function $a(x) - b(x)$.

Solution: The horizontal intercepts of the function $a(x) - b(x)$ are given by the two points of intersection of the graphs of $y = a(x)$ and $y = b(x)$. One of these occurs at $x \approx -2.7$ and the other at $x \approx 2.7$.

Horizontal intercept(s): $x \approx -2.7, 2.7$

d. [4 points] Find a possible formula for the polynomial $a(x)$. You do not need to simplify your answer. *Show your work.*

Solution: Note that the long-run behavior of the graph shows that the degree of $a(x)$ is even, and since the graph "turns" three times, the degree must be at least four.
The polynomial $a(x)$ has zeros at $x = -3$ and $x = 0$ and a zero of even multiplicity at $x = 2$. A possible formula for $a(x)$ is then $a(x) = k(x+3)(x)(x-2)^2$. Since the point $(-2, 8)$ is on the graph of $y = a(x)$, we see that $a(-2) = 8$, so $8 = k(-2+3)(-2)(-2-2)^2$. Thus $8 = k(-32)$ so $k = -1/4$. Hence a possible formula for $a(x)$ is $a(x) = -\frac{1}{4}x(x+3)(x-2)^2$.

$a(x) = -\dfrac{1}{4}x(x+3)(x-2)^2$