

9. [7 points] In the United States, the number of werewolves, W , living in a given state is a function $W = g(V)$ of the number of vampires, V , that live in that state. The formula for $g(V)$ is $g(V) = kV^{2/3}$, where k is a positive constant. The constant k does not depend on the state.
- a. [4 points] In Pennsylvania, there are 1728 vampires and 720 werewolves. In Indiana, there are 512 vampires. How many werewolves live in Indiana?

Solution: The data from Pennsylvania show that $g(1728) = 720$. Therefore, $1720 = k(1728)^{2/3}$, so $720 = k144$ and $k = 1720/144 = 5$. Thus, the formula for $g(V)$ is $g(V) = 5V^{2/3}$. Since there are 512 vampires in Indiana, we compute that the number of werewolves in Indiana is $g(512) = 5(512^{2/3}) = 320$.

Answer: 320 werewolves

- b. [3 points] There are 50% more vampires in Ohio than there are in Michigan. How much larger is the werewolf population of Ohio than that of Michigan?
Your answer should be accurate to at least 0.01%.

Solution: Let V_M be the number of vampires in Michigan. Then the number of vampires in Ohio is $1.5V_M$. So the number of werewolves in Michigan is $kV_M^{2/3}$ and the number of werewolves in Ohio is $k(1.5V_M)^{2/3}$. Hence the ratio of the number of werewolves in Ohio to the number of werewolves in Michigan is

$$\frac{k(1.5V_M)^{2/3}}{kV_M^{2/3}} = \frac{(1.5)^{2/3}V_M^{2/3}}{V_M^{2/3}} = 1.5^{2/3} \approx 1.31037.$$

Hence, there are approximately 31.04% more werewolves in Ohio than there are in Michigan.

Answer: The werewolf population of Ohio is 31.04% percent larger than the werewolf population of Michigan.