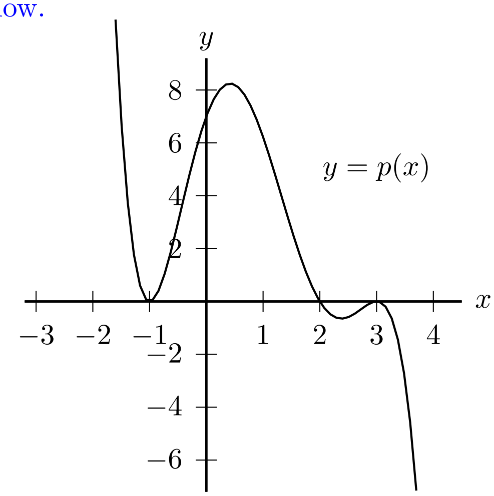
9. [5 points] Find a formula for *one* polynomial $p(x)$ that satisfies *all* of the following conditions.

- The vertical intercept of the graph of $p(x)$ is 7.
- The graph of $p(x)$ has horizontal intercepts -1, 2, and 3 (and no others).
- $\lim_{x\to\infty} p(x) = -\infty$ and $\lim_{x\to-\infty} p(x) = \infty$.
- The degree of $p(x)$ is at most 6.

Show your work and reasoning carefully. You might find it helpful to first sketch a graph. There may be more than one possible answer, but you should give only one answer.

Solution:

A graph showing such a polynomial is shown below.



To find a formula, note that the factored form of such a polynomial could be

$$p(x) = a(x+1)^2(x-2)(x-3)^2.$$

The vertical intercept is given by $p(0)$ so we use the fact that $p(0) = 7$ to determine a. That is

$$7 = p(0) = a(0+1)^2(0-2)(-3)^2 = a(-18),$$

so $a = -\frac{7}{18}$. Hence a possible formula is $p(x) = -\frac{7}{18}(x+1)^2(x-2)(x-3)^2$.

Note: Another possibility (corresponding to a double root at $x = 3$ rather than at $x = 2$) would be $p(x) = -\frac{7}{12}(x+1)^2(x-2)^2(x-3)$

$p(x) =$ $-\frac{7}{18}(x+1)^2(x-2)(x-3)^2$ **or** $-\frac{7}{12}(x+1)^2(x-2)^2(x-3)$

10. [4 points] If $K = G(t) = \dfrac{e^t+3}{7+e^t}$ find a formula for $G^{-1}(K)$.

Solution: To find a formula for $G^{-1}(K)$ we need to solve for t in the equation $K = \dfrac{e^t+3}{7+e^t}$.

$$\begin{aligned} K &= \frac{e^t+3}{7+e^t} \\ K(7+e^t) &= e^t+3 \\ 7K + Ke^t &= e^t + 3 \\ Ke^t - e^t &= 3 - 7K \\ e^t(K-1) &= 3-7K \\ e^t &= \frac{3-7K}{K-1} \\ t &= \ln\left(\frac{3-7K}{K-1}\right) \end{aligned}$$

Answer: $G^{-1}(K) =$ $\ln\left(\dfrac{3-7K}{K-1}\right)$