9. [5 points] Find a formula for one polynomial $p(x)$ that satisfies all of the following conditions.

- The vertical intercept of the graph of $p(x)$ is 7.
- The graph of $p(x)$ has horizontal intercepts $-1, 2, 3$ (and no others).
- $\lim_{x \to \infty} p(x) = -\infty$ and $\lim_{x \to -\infty} p(x) = \infty$.
- The degree of $p(x)$ is at most 6.

Show your work and reasoning carefully. You might find it helpful to first sketch a graph. There may be more than one possible answer, but you should give only one answer.

**Solution:**

To find a formula, note that the factored form of such a polynomial could be

$$p(x) = a(x + 1)^2(x - 2)(x - 3)^2.$$ 

The vertical intercept is given by $p(0)$ so we use the fact that $p(0) = 7$ to determine $a$. That is

$$7 = p(0) = a(0 + 1)^2(0 - 2)(-3)^2 = a(-18),$$

so $a = -\frac{7}{18}$. Hence a possible formula is

$$p(x) = -\frac{7}{18}(x + 1)^2(x - 2)(x - 3)^2.$$ 

Note: Another possibility (corresponding to a double root at $x = 3$ rather than at $x = 2$) would be $p(x) = -\frac{7}{12}(x + 1)^2(x - 2)(x - 3)^2$.

10. [4 points] If $K = G(t) = \frac{e^t + 3}{7 + e^t}$ find a formula for $G^{-1}(K)$.

**Solution:** To find a formula for $G^{-1}(K)$ we need to solve for $t$ in the equation $K = \frac{e^t + 3}{7 + e^t}$.

$$K = \frac{e^t + 3}{7 + e^t}$$

$$K(7 + e^t) = e^t + 3$$

$$7K + Ke^t = e^t + 3$$

$$Ke^t - e^t = 3 - 7K$$

$$e^t(K - 1) = 3 - 7K$$

$$e^t = \frac{3 - 7K}{K - 1}$$

$$t = \ln \left(\frac{3 - 7K}{K - 1}\right)$$

**Answer:** $G^{-1}(K) = \ln \left(\frac{3 - 7K}{K - 1}\right)$