11. [8 points] Every morning, a student gets a cup of coffee from a local coffee shop and then sits down to work. Today the coffee was served at a temperature of 185°F. Let $C(t)$ be the temperature, in degrees Fahrenheit, of the cup of coffee t hours after it was poured today, and let $D(t) = C(t) - 70$.

Throughout this problem, show your work carefully and give all answers in exact form or accurate to at least three decimal places.

a. [1 point] Find $D(0)$.

Solution: $D(0) = C(0) - 70 = 185 - 70 - 115$.

Answer: $D(0) =$ 115

b. [2 points] $D(t)$ is an exponential function with a *continuous* hourly decay rate of 80%. Find a formula for $D(t)$ and then find a formula for $C(t)$

Solution: In part (a), we found that the initial value of D is 115, so $D(t) = 115e^{-0.8t}$. Since $D(t) = C(t) - 70$ we have $C(t) = 70 + D(t) = 70 + 115e^{-0.8t}$.

$D(t) =$ $115e^{-0.8t}$ $C(t) =$ $70 + 115e^{-0.8t}$

c. [1 point] By what percent does $D(t)$ decrease each hour?

Solution: The hourly decay factor is $b = e^{-0.8} \approx 0.44933$ so $D(t)$ decreases by about 55.067% per hour.

Answer: **by about** 55.067%

d. [2 points] By how many degrees did the temperature of the cup of coffee decrease within the first 30 minutes after it was poured?

Solution: The temperature, in degrees Fahrenheit, of the coffee when it was poured was $C(0) = 185$ and its temperature (in °F) 30 minutes after it was poured was

$$C(0.5) = 70 + 115e^{-0.8(0.5)} \approx 147.09.$$

So the temperature of the coffee decreased by

$$C(0) - C(0.5) = 185 - (70 + 115e^{-0.8(0.5)}) \approx 37.91°\text{F}$$

within the first 30 minutes after it was poured.

Answer: **by about** 37.91°**F**

e. [2 points] Find and interpret, in the context of this problem, any horizontal asymptotes of the function $C(t)$.

Solution: $D(t)$ is an exponentially decreasing positive function so it has horizontal asymptote $y = 0$ ($D(t) \to 0$ as $t \to \infty$). $C(t) = D(t) + 70$ so its graph is obtained from the graph of $D(t)$ by shifting up 70 units. Hence the graph of $y = C(t)$ has horizontal asymptote $\boxed{y = 70}$ (and $C(t) \to 70$ as $t \to \infty$).
Interpretation: As time passes, the temperature of the coffee approaches 70°F. (This is probably the air temperature in the coffee shop.)