

4. [12 points]

Figure 1 below gives some data for an invertible function f and Figure 2 shows the entire graph of a function g . Use this information to answer the questions below.

x	0	1	2	3	4	5	6
$f(x)$	2	6	5	4	1	3	7

Figure 1

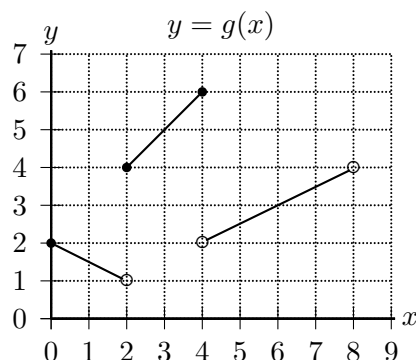


Figure 2

- a. [3 points] What is the domain of g ? What is the domain of g^{-1} ?
(Use either inequalities or interval notation to give your answers.)

Domain of g : $[0, 8]$ Domain of g^{-1} : $(1, 6]$

- b. [4 points]

i. Evaluate $3f(2) + 1$.

Solution: $3f(2) + 1 = 3(5) + 1 = 16$.

Answer: 16

ii. Evaluate $g(g(4))$.

Solution: $g(g(4)) = g(6) = 3$.

Answer: 3

iii. Evaluate $g(f(1) - 1)$.

Solution: $g(f(1) - 1) = g(6 - 1) = g(5) = 2.5$.

Answer: 2.5

iv. Evaluate $f^{-1}(g^{-1}(3))$.

Solution: $f^{-1}(g^{-1}(3)) = f^{-1}(6) = 1$.

Answer: 1

- c. [2 points] Find the average rate of change of $f(x)$ between $x = 2$ and $x = 5$.

Solution: The average rate of change of $f(x)$ between $x = 2$ and $x = 5$ is

$$\frac{f(5) - f(2)}{5 - 2} = \frac{3 - 5}{3} = -\frac{2}{3}.$$

Answer: $-\frac{2}{3}$

- d. [3 points] Suppose $h(x) = 3 + 4x$. What transformations must be performed on the graph of $y = g(x)$ to obtain the graph of $y = h(g(x))$?
(Be specific and give the transformations in the appropriate order.)

Solution: We have $h(g(x)) = 3 + 4g(x)$, so we can obtain the graph of $h(g(x))$ from the graph of $g(x)$ by first stretching vertically by a factor of 4 and then shifting the resulting graph up 3 units. (Alternatively, we could first shift up $3/4$ of a unit and then stretch the resulting graph vertically by a factor of 4.)