5. [9 points] A diver jumps up off of a diving board into a swimming pool below. Until the moment the diver enters the water, his height above the water (measured in feet) t seconds after his feet leave the diving board is $h(t) = -16t^2 + 8t + 10$.
Throughout this problem, remember to show your work and reasoning.
Give your answers in exact form or accurate to at least three decimal places.

a. [3 points] Use the method of completing the square to rewrite the formula for $h(t)$ in vertex form. (*Carefully show your work step-by-step.*)

Solution: Applying the method of completing the square, we have

$$h(t) = -16t^2 + 8t + 10 = -16\left(t^2 - \frac{1}{2}t\right) + 10$$
$$= -16\left(t^2 - \frac{1}{2}t + \left(-\frac{1}{4}\right)^2 - \left(-\frac{1}{4}\right)^2\right) + 10 = -16\left(\left(t - \frac{1}{4}\right)^2 - \frac{1}{16}\right) + 10$$
$$= -16\left(t - \frac{1}{4}\right)^2 + 1 + 10 = -16\left(t - \frac{1}{4}\right)^2 + 11$$

Answer: $h(t) = -16\left(t - \frac{1}{4}\right)^2 + 11$

b. [2 points] After how many seconds does the diver reach his maximum height above the pool? What is that maximum height?

Solution: Based on the vertex form found in part (a), the vertex of the graph of $h(t)$ (which is a parabola) is $(1/4, 11)$. Since the leading coefficient is negative, the parabola opens downward and this vertex gives the maximum of $h(t)$.

After 0.25 seconds, the diver reaches his maximum height of 11 feet.

c. [2 points] After how many seconds does the diver enter the water?

Solution: We must solve the equation $h(t) = 0$. Using our vertex form from part (a), we have

$$-16\left(t - \frac{1}{4}\right)^2 + 11 = 0 \quad \text{so} \quad -16\left(t - \frac{1}{4}\right)^2 = -11 \quad \text{and} \quad \left(t - \frac{1}{4}\right)^2 = \frac{11}{16}.$$
$$\text{Hence} \quad t - \frac{1}{4} = \pm\sqrt{\frac{11}{16}} = \pm\frac{\sqrt{11}}{4} \quad \text{so} \quad t = \frac{1 \pm \sqrt{11}}{4}.$$

Note that $\frac{1 - \sqrt{11}}{4} < 0$ so $\frac{1 + \sqrt{11}}{4} \approx 1.0792$ is the solution corresponding to a time after the diver left the diving board. (Alternatively, we could apply the quadratic formula to the original formula for $h(t)$ or use a graphing calculator to approximate the positive zero of $h(t)$.)

The diver enters the water $\frac{1 + \sqrt{11}}{4}$ seconds after his feet leave the diving board.

d. [2 points] In the context of this problem, what are the domain and range of $h(t)$?
(*Use either inequalities or interval notation to give your answers.*)

Domain: $\left[0, \frac{1 + \sqrt{11}}{4}\right]$ **Range:** $[0, 11]$