6. [9 points] The tables below provide data from three functions, f, g, and h. Each of these functions is either a *linear* function, an *exponential* function, a *sinusoidal* function, or a *power* function. (Note that there may be either zero, one, or more than one function of each type.)

x	-2	-1	1	2
f(x)	12	1.5	-1.5	-12

x	-3	-1	1	3
g(x)	12	6.5	1	-4.5

x	1	3	5	7
h(x)	32.4	10.8	3.6	1.2

a. [3 points] What type of function is f? (*Circle* ONE *answer.*)

linear exponential sinusoidal [power]

Find a formula for $f(x)$. (*Show your work carefully and use* exact form.)

Solution: $f(x)$ is a power function so there are constants k and p so that $f(x) = kx^p$. Note that $f(x) = -1.5$, so $-1.5 = k(1)^p = k(1) = k$. So $f(x) = -1.5x^p$. Now $f(2) = -12$ so $-12 = -1.5(2)^p$, so $8 = 2^p$ and $p = 3$. Thus a formula for $f(x)$ is $f(x) = -1.5x^3$.

Answer: $f(x) =$ $-1.5x^3$

b. [3 points] What type of function is g? (*Circle* ONE *answer.*)

[linear] exponential sinusoidal power

Find a formula for $g(x)$. (*Show your work carefully and use* exact form.)

Solution: We see that $g(x)$ is linear by noting that the average rate of change between each pair of consecutive inputs is constant (-2.75). $g(x)$ is linear with (constant) average rate of change equal to -2.75. Using point-slope form (with the point $(1,1)$) we find that $g(x) - 1 = -2.75(x-1)$ so $g(x) = 1 - 2.75(x-1)$. (This reduces to $g(x) = 3.75 - 2.75x$ in slope-intercept form.)

Answer: $g(x) =$ $1 - 2.75(x-1) = 3.75 - 2.75x$

c. [3 points] What type of function is h? (*Circle* ONE *answer.*)

linear [exponential] sinusoidal power

Find a formula for $h(x)$. (*Show your work carefully and use* exact form.)

Solution: (*)Note that the ratio of consecutive outputs is constant (1/3) and the the difference between consecutive inputs is also constant (2).
Hence $h(x)$ appears to be exponential. There are constants a and b so that $h(x) = ab^x$. Using the facts that $h(1) = 32.4$ and $h(3) = 10.8$ we have $ab = 32.4$ and $ab^3 = 10.8$. Hence, we see that $\frac{ab^3}{ab} = \frac{10.8}{32.4}$ so $b^2 = \frac{1}{3}$. (This is what we had already determined in (*) above.) Thus $b = \sqrt{\frac{1}{3}}$ so $a\sqrt{\frac{1}{3}} = 32.4$ and $a = \frac{32.4}{\sqrt{\frac{1}{3}}} = 32.4\sqrt{3}$.

Answer: $h(x) =$ $32.4\sqrt{3}\left(\frac{1}{\sqrt{3}}\right)^x = 32.4\sqrt{3}(3^{-x/2}) = 32.4(3)^{(1-x)/2}$