9. [5 points] Find a formula for one polynomial \( p(x) \) that satisfies all of the following conditions.

- The vertical intercept of the graph of \( p(x) \) is 7.
- The graph of \( p(x) \) has horizontal intercepts \(-1, 2, \) and \(3 \) (and no others).
- \( \lim_{x \to \infty} p(x) = -\infty \) and \( \lim_{x \to -\infty} p(x) = \infty. \)
- The degree of \( p(x) \) is at most 6.

Show your work and reasoning carefully. You might find it helpful to first sketch a graph.

**Solution:**

A graph showing such a polynomial is shown below.

To find a formula, note that the factored form of such a polynomial could be

\[
p(x) = a(x + 1)^2(x - 2)(x - 3)^2.
\]

The vertical intercept is given by \( p(0) \) so we use the fact that \( p(0) = 7 \) to determine \( a \). That is

\[
7 = p(0) = a(0 + 1)^2(0 - 2)(-3)^2 = a(-18),
\]

so \( a = \frac{7}{18} \). Hence a possible formula is

\[
p(x) = -\frac{7}{18}(x + 1)^2(x - 2)(x - 3)^2 \text{ or } -\frac{7}{12}(x + 1)^2(x - 2)^2(x - 3).
\]

10. [4 points] If \( K = G(t) = \frac{e^t + 3}{7 + e^t} \) find a formula for \( G^{-1}(K) \).

**Solution:** To find a formula for \( G^{-1}(K) \) we need to solve for \( t \) in the equation \( K = \frac{e^t + 3}{7 + e^t} \).

\[
K = \frac{e^t + 3}{7 + e^t}
\]

\[
K(7 + e^t) = e^t + 3
\]

\[
7K + Ke^t = e^t + 3
\]

\[
Ke^t - e^t = 3 - 7K
\]

\[
e^t(K - 1) = 3 - 7K
\]

\[
e^t = \frac{3 - 7K}{K - 1}
\]

\[
t = \ln \left( \frac{3 - 7K}{K - 1} \right)
\]

**Answer:** \( G^{-1}(K) = \ln \left( \frac{3 - 7K}{K - 1} \right) \)