8. [11 points] On the beaches of Mexico, there is a population of picky snails that wait for special shells to wash up onto the shore. These snails can only live in these particular shells, as the snails have become accustomed to the comfort in these shells.

Suppose the number of hundreds of special shells on the beaches of Mexico t years after the beginning of 2013 is $h(t) = (t^2 + 7)(4t - 7)^2$

and the population, in hundreds, of picky snails t years after the beginning of 2013 is

$$p(t) = (t-2)^2(8t^2 + 30).$$

Throughout this problem, remember to clearly show your work and reasoning.

a. [3 points] Find the leading term and any zeros of h(t). If appropriate, write "NONE" in the answer blank provided.

Answers: Leading Term: _____ Zero(s): _____

b. [3 points] The number of shells per snail is $Q(t) = \frac{h(t)}{p(t)}$.

Find the equations of all vertical asymptotes ("V.A.") and horizontal asymptotes ("H.A.") of the graph of y = Q(t). If appropriate, write "NONE" in the answer blank provided.

Answers: V.A.: ______ H.A.: ____

There is a competitive population of crabs that live on the same beaches. Suppose that there are 1200 of these crabs at the beginning of 2013, and that the population grows at a *continuous* annual rate of 35%. Let c(t) be the population, in hundreds, of these crabs t years after the beginning of 2013.

c. [2 points] Find a formula for c(t).

Answer: $c(t) = \underline{\hspace{1cm}}$

d. [3 points] The crabs like the same special shells as the snails do. Write a formula for the ratio of the number of shells to the number of crabs t years after the beginning of 2013.

Answer:

In the long run, what happens to the ratio of the number of shells to the number of crabs? In other words, assuming the functions described in this problem continue to be accurate models, what happens to this ratio after many, many years?

You must clearly indicate your reasoning in order to receive any credit for this problem.