

1. [10 points] Foghorn is a chicken that is learning how to fly. In fact, he trains every day by jumping off the top of his coop and flapping his wings. Today, his height above the ground, in feet, t seconds after jumping is given by the function $h(t) = -16t^2 + 20t + 6$.

Note that once he lands on the ground, he stays on the ground.

- a. [2 points] How long after Foghorn jumps off his coop does he hit the ground?

Be sure to show your work and give your final answer in exact form.

Solution: To find when Foghorn hits the ground, we set $h(t) = 0$ and solve for t . Using the quadratic formula, we find

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-16)(6)}}{2(-16)} = \frac{-20 \pm \sqrt{784}}{-32} = \frac{-20 \pm 28}{-32}$$

So, $t = -0.25$ and $t = 1.5$. Because he starts on top of the coop, $t = -0.25$ does not make any sense as an answer. So, Foghorn hits the ground 1.5 seconds after jumping off the coop.

Answer: 1.5 seconds

- b. [4 points] Use the method of completing the square to put the formula for $h(t)$ into vertex form. *Carefully show your algebraic work step-by-step.*

Solution:

$$\begin{aligned} h(t) &= -16t^2 + 20t + 6 = -16\left(t^2 - \frac{5}{4}t\right) + 6 = -16\left(t^2 - \frac{5}{4}t + \left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2\right) + 6 \\ &= -16\left(\left(t - \frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2\right) + 6 = -16\left(t - \frac{5}{8}\right)^2 + 16\left(\frac{5}{8}\right)^2 + 6 = -16\left(t - \frac{5}{8}\right)^2 + 12.25 \end{aligned}$$

Answer: $h(t) =$ $-16\left(t - \frac{5}{8}\right)^2 + 12.25$

- c. [2 points] What is the maximum height Foghorn reaches? **Answer:** 12.25 feet

When does he reach his maximum height? **Answer:** 5/8 seconds after he jumps

Solution: The vertex form gives a maximum height of 12.25 feet 5/8 seconds after he jumps.

- d. [2 points] What are the domain and range of $h(t)$ in the context of this problem?

Use either interval notation or inequalities to give your answers.

Solution: In the context of this problem, the domain is from when he jumps to when he lands. The domain is then $[0, 1.5]$. The range is from the ground, 0 feet, to the maximum height he reaches, which is 12.25 feet. The range is then $[0, 12.25]$.

Answers: Domain: $[0, 1.5]$ Range: $[0, 12.25]$