

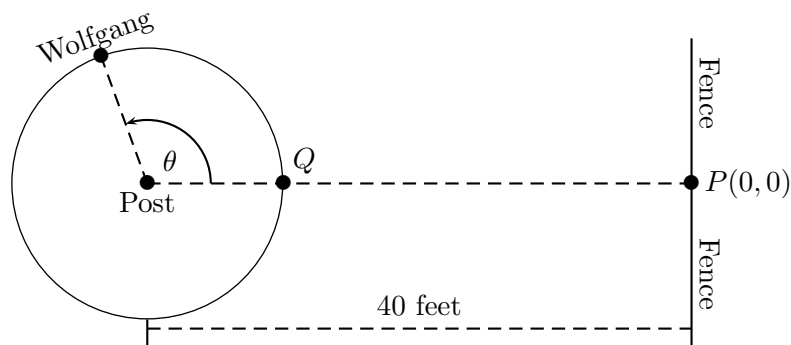
10. [4 points] Suppose that the number of acorns in Squishy squirrel's nest is proportional to the cube of the number of squirrels currently living there. If there are 113 acorns in his nest when there are two squirrels living there, how many acorns will there be in Squishy's nest when there are four squirrels living there? *Remember to show your work carefully.*

Solution: Let A be the number of acorns in Squishy squirrel's nest. Let Q be the number of squirrels living there. Then we know that $A = kQ^3$ where k is some constant. If there are 113 acorns in his nest when there are two squirrels living there, then $113 = k(2^3)$ so $k = \frac{113}{8}$ and a general formula is $A = \frac{113}{8}Q^3$.

Therefore, when there are four squirrels living in the nest, $A = \frac{113}{8}(4^3) = \frac{113}{8}(64) = 113(8) = 904$. So there are 904 acorns in Squishy squirrel's nest when there are four squirrels living there.

Answer: 904 acorns

11. [7 points] Wolfgang the wolf is on a 10-foot long leash that is tied to a post that is 40 feet west of a fence.



Because he dislikes being on his leash, he stays 10 feet away from the post at all times.

- a. [4 points] Suppose we think of the origin at the point P as shown in the diagram and that the unit of measurement is feet so that the coordinates of the post are $(-40, 0)$. Find Wolfgang's coordinates when he is at the angle θ shown in the diagram. (*Your answer should be in terms of θ .*)

Solution: If the post were at the origin, his coordinates would be $(10 \cos(\theta), 10 \sin(\theta))$. Since the post is 40 feet to the left of the origin, his first coordinate will be 40 units less, i.e. $-40 + 10 \cos(\theta)$. This does not change the second coordinate.

Answer: Wolfgang's coordinates are $\left(\underline{-40 + 10 \cos(\theta)}, \underline{10 \sin(\theta)} \right)$.

- b. [3 points] Wolfgang starts walking counterclockwise from the point Q . The angle θ through which Wolfgang has walked is a function of the amount of time he has been walking. Let $\theta = z(t)$ be the angle (in radians) through which Wolfgang has walked after he has been walking for t minutes. Let $A(t)$ be the distance Wolfgang has traveled along the circle in t minutes. Find a function $f(t)$ such that $A(t) = f(z(t))$.

Solution: Using the arclength formula, $A(t) = 10\theta = 10z(t)$. We want to find a function $f(t)$ such that

$$f(z(t)) = A(t) \quad \text{i.e. so that } f(z(t)) = 10z(t).$$

The above shows that if the function f is given an input of $z(t)$, the output is $10z(t)$. Thus, f should be a function that takes its input and multiplies it input by 10. So $f(t) = 10t$.

Answer: $f(t) = \underline{10t}$