12. [10 points] Consider the functions f, g, and h defined as follows:

$$f(x) = a + bx \qquad g(x) = cx^d \qquad h(x) = w(1+r)^x$$

for nonzero constants a, b, c, d, r, and w with $r > -1$.

For each of the questions below, circle all the correct answers from among the choices provided, or circle NONE OF THESE if appropriate.

a. [2 points] The graph of which function(s) definitely has at least one horizontal intercept?

$\boxed{f(x)}$ $g(x)$ $h(x)$ NONE OF THESE

Solution: $f(x)$ will always have a horizontal intercept since it is linear with nonzero slope.
If $d < 0$, e.g. if $g(x) = 10x^{-1}$, then $g(x)$ does not have a horizontal intercept.
$h(x)$ does not have a horizontal intercept because it is an exponential function and w is nonzero.

b. [2 points] The graph of which function(s) definitely has at least one horizontal asymptote?

$f(x)$ $g(x)$ $\boxed{h(x)}$ NONE OF THESE

Solution: $f(x)$ is a linear function with nonzero slope so does not have a horizontal asymptote.
If $d > 0$, e.g. if $g(x) = 10x^2$, then it does not have a horizontal asymptote.
$y = h(x)$ is an exponential function so has horizontal asymptote $y = 0$.

c. [2 points] Which function(s) is(are) definitely invertible?

$\boxed{f(x)}$ $g(x)$ $\boxed{h(x)}$ NONE OF THESE

Solution: The linear function with nonzero slope, $f(x)$, and the exponential function, $h(x)$ are definitely invertible. (They pass the horizontal line test, for example.)
$g(x)$ may or may not be invertible. For example, if $g(x) = 10x^2$, then it is not invertible.

d. [2 points] How many times could the graph of $f(x)$ intersect the graph of $h(x)$?

$\boxed{0}$ $\boxed{1}$ $\boxed{2}$ 3 4 more than 4

Solution: A linear function can intersect an exponential function either 0, 1, or 2 times. For example, $y = x + 1$ and $y = 2(1+3)^x$ do not intersect, whereas $y = -x + 1$ and $y = 2(1+3)^x$ intersect exactly once, and $y = x + 5$ and $y = 2(1+3)^x$ intersect exactly two times. (They cannot intersect more than two times due to their long-run behavior.)

e. [2 points] Suppose the graph of h is concave up. Which of the following is(are) definitely true?

$\boxed{w > 0}$ $w < 0$ $r > 0$ $r < 0$ NONE OF THESE

Solution: An exponential function is concave up if and only if its initial value is positive.
That is, whether $-1 < r < 0$ or $r > 0$, if $w > 0$, then the graph of h will be concave up whereas if $w < 0$, the graph of h will be concave down.