3. [10 points] Let $G(v)$ be the number of minutes it takes Goober the gorilla to eat a meal consisting of v pounds of vegetation.

a. [2 points] Suppose b and n are positive constants.
Give a practical interpretation of the equation $G^{-1}(b) = n$ in the context of this problem. *Use a complete sentence and include units.*

Solution: It takes Goober b minutes to eat a meal consisting of n pounds of vegetation.

b. [4 points] Suppose that there are positive constants c and d so that a formula for $G(v)$ is given by

$$G(v) = cv^d.$$

If $G(2) = 9$ and $G(3) = 18$, find the *exact* values of the constants c and d.

Solution: $G(2) = 9$ and $G(3) = 18$, so $c(2)^d = 9$ and $c(3)^d = 18$. Taking ratios, we find

$$\frac{c(3)^d}{c(2)^d} = \frac{18}{9}$$
$$\frac{3^d}{2^d} = 2$$
$$\left(\frac{3}{2}\right)^d = 2$$
$$1.5^d = 2$$

Using logarithms:

$$\ln(1.5^d) = \ln(2)$$
$$d\ln(1.5) = \ln(2)$$
$$d = \frac{\ln(1.5)}{\ln(2)}$$

Substituting into the equation $c(2)^d = 9$, we find $c(2)^{\ln(1.5)/\ln(2)} = 9$ so $c = \dfrac{9}{2^{\ln(1.5)/\ln(2)}}$.

Answers: $c =$ $\dfrac{9}{2^{\ln(1.5)/\ln(2)}}$ and $d =$ $\dfrac{\ln(1.5)}{\ln(2)}$

c. [4 points] Suppose that the number of minutes it takes Goober's friend Toober to eat a meal consisting of v pounds of vegetation is $m = T(v)$, which is given by the formula

$$T(v) = q + \frac{\ln(v+2)}{\ln(5)}$$

for some constant q. Find a formula for $T^{-1}(m)$. *Show your work carefully.*
Note that your answer should be in exact form and be given in terms of m and q.

Solution: To find $T^{-1}(m)$, we solve for v in the equation $m = q + \dfrac{\ln(v+2)}{\ln(5)}$.

$$m = q + \frac{\ln(v+2)}{\ln(5)}$$
$$m - q = \frac{\ln(v+2)}{\ln(5)}$$
$$(\ln(5))(m-q) = \ln(v+2)$$

Exponentiating:

$$e^{\ln(5)(m-q)} = v + 2$$
$$e^{\ln(5)(m-q)} - 2 = v$$

Thus $T^{-1}(m) = e^{\ln(5)(m-q)} - 2$.

Note that $e^{\ln(5)(m-q)} = \left(e^{\ln(5)}\right)^{m-q} = 5^{m-q}$ so we can simplify to $T^{-1}(m) = 5^{m-q} - 2$.

Answer: $T^{-1}(m) =$ $5^{m-q} - 2$ **(or** $e^{\ln(5)(m-q)} - 2$**)**