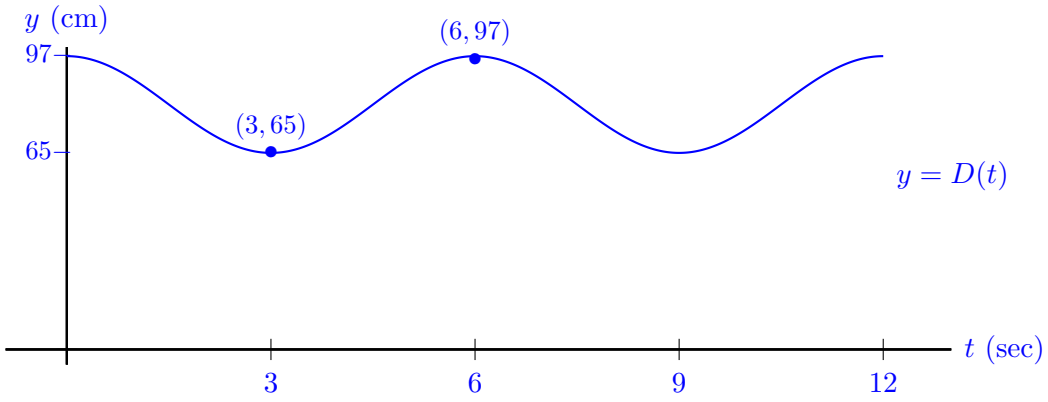
4. [13 points] Severus Snake is slithering along the banks of a river. At noon, a scientist starts to track Severus's distance away from the edge of the river. After a few minutes, the scientist realizes Severus's distance away from the edge of the river is a sinusoidal function. Let $D(t)$ be Severus's distance, in centimeters, away from the edge of the river t seconds after noon.

a. [5 points] At noon exactly, the scientist notes that Severus is 97 centimeters away from the edge of the river, which is the farthest away he ever gets. Three seconds after that, Severus's distance is 65 centimeters away from the river, the closest he gets.
Graph $y = D(t)$ for $0 \le t \le 12$. (*Clearly label the axes and important points on your graph. Be very careful with the* **shape and key features of your graph.**)



b. [6 points] Find the period, amplitude, equation of the midline, and a formula for the sinusoidal function $D(t)$. (*Include units for the period and amplitude.*)

Solution: A maximum and succeeding minimum of a sinusoidal function occur half a period apart. In this case, they occur 3 seconds apart, so the period is 6 seconds.
The midline is found by averaging the maximum and minimum values. This gives $y = \frac{97+65}{2} = 81$.
The amplitude is the distance between a maximum or minimum and the midline which is $|97 - 81| = 16$ centimeters.
Because the maximum occurs at noon ($t = 0$), the cosine function is a good candidate to use here. Let $D(t) = A\cos(B(t-h)) + k$. Then we have $A = 16$, $B = \frac{2\pi}{6} = \frac{\pi}{3}$, and $k = 81$. So, we have $D(t) = 16\cos\left(\frac{\pi}{3}(t-h)\right) + 81$. Because the maximum occurs at $t = 0$, we do not need a horizontal (or phase) shift. Thus, one possible formula for $D(t)$ is $D(t) = 16\cos\left(\frac{\pi t}{3}\right) + 81$.

Period: 6 **seconds**

Amplitude: 16 **centimeters**

Midline: $y = 81$

Formula: $D(t) =$ $16\cos(\frac{\pi}{3}t) + 81$

c. [2 points] How far away from the river is Severus 11 seconds after noon?
Give your answer accurate to at least two decimal places.

Solution: Severus is $D(11)$ centimeters away from the river 11 seconds after noon. Using the formula from above, $D(11) = 16\cos\left(\frac{11\pi}{3}\right) + 81 = 16\left(\frac{1}{2}\right) + 81 = 8 + 81 = 89$. So, Severus is 89 centimeters away from the river 11 seconds after noon.

Answer: 89 **centimeters**